

Screened potential and the baryon spectrum

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Abstract

We show that in a quark model scheme the use of a screened potential, suggested by lattice QCD, instead of an infinitely rising one with the interquark distance, provides a more adequate description of the high-energy baryon spectrum. In particular an almost perfect parallelism between the predicted and observed number of states comes out throwing new light about the so-called missing resonance problem.

Keywords: nonrelativistic quark models, baryon spectrum, missing states

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I. INTRODUCTION

Constituent quark models of baryon structure are based on the assumption of effective quark degrees of freedom so that a baryon is a three-quark color-singlet state. Quarks are bound by means of a phenomenological qq potential attempting to incorporate observed features of the hadron spectra. Although not rigorously proven from QCD, numerical simulations for heavy quarks in the lattice in the quenched approximation (considering only valence quarks) show out a qq confining potential linearly rising with the interquark distance [1]. This potential produces an infinite discrete hadron spectrum. The implementation of a confining force of this type with effective one gluon and/or boson exchanges, or other effective interactions, turns out to be fruitful in the construction of quark potential models that provide with a precise description of baryon spectroscopy [2–5]. However an outstanding problem remains unsolved: all models predict a proliferation of baryon states at excitation energies above 1 GeV which are not experimentally observed as resonances. This difference between the quark model prediction and the data about the number of physical resonances is known as the missing resonance problem. An explanation for such a discrepancy has been given by realizing that most (but not all) of the missing states could be too inelastic to be easily observed [6,7]. To test experimentally the situation is a current objective of the CLAS collaboration in TJNAF [8] and also the BES collaboration in Beijing [9].

In this article we propose to revisit the baryon spectrum centering the attention in its high-energy excited states. As this sector in spectroscopic quark models is essentially determined by the linear confinement, we are driven to refine the potential. To do this we incorporate effects associated to the creation of light $q\bar{q}$ pairs out of vacuum. For heavy quarks the spontaneous pair creation between them may give rise to a breakup of the color flux tube in such a way that the quark-quark potential does not rise with the interquark distance but it reaches a maximum saturation value. Though from previous lattice calculations a potential parametrization of these screening effects was proposed [10] there is not a convincing confirmation of string breaking [11]. However a quite rapid cross-over from a linear to a flat potential is well established in SU(2) Yang-Mills theory [12]. We shall assume hereforth that such screening occurs in QCD and can be approximately parametrized, even for light quarks, through a potential as proposed in Ref. [10]. Such an assumption has been employed in the literature to study the nonstrange baryon spectrum [13], the heavy meson spectrum [14], and as a possible explanation of the nonlinear hadronic Regge trajectories [15]. For the baryon spectrum an improvement in the description of the higher energy excitations with respect to the linear confining potential case was obtained. For the heavy mesons a pretty good description of bottomonium was accomplished. Nevertheless the more striking predictions associated to screening, say the breaking of the color flux tube between quarks (or quark and antiquark) and the corresponding finiteness of the bound state spectrum could not be tested in the meson sector because of the lack of sufficient data at the high-energy excitation region. On the other hand in Ref. [13] the results for the baryon sector, obtained by means of a variational method with gaussian trial wave functions, were restricted to $J \leq 7/2$ and no threshold stability analysis was carried out. Our hope here is that the more extensive and deep analysis of the known nonstrange high-energy baryon spectrum may allow us to establish, or at least make feasible, the validity of such predictions.

To this purpose we start in Sec. II by revising the nonstrange baryon spectrum obtained

with a minimal model that includes an effective linear confinement plus a one-gluon-exchange (OGE) like potential. From it we establish the connected missing resonance problem. Then in Sec. III we introduce screening effects in the potential and derive the value of the parameters by plausibility arguments and fitting to data. Sec. IV is devoted to the results we obtain for the whole spectrum and their comparison to data. Finally in Sec. V we summarize our work and resume our main conclusions.

II. STRICT CONFINEMENT

Lattice calculations in the quenched approximation derive, for heavy quarks, a confining interaction linearly dependent on the interquark distance [1]. This form of strict confinement has been widely used for light quarks when studying the baryon spectrum within a quark model framework [2–5]. To illustrate our discussion and for the sake of simplicity we shall make use of a nonrelativistic quark model potential containing besides the linear confinement a minimal OGE term (Coulomb and spin-spin). Such a model was proposed in Ref. [16] to describe the meson spectrum and it was later on applied to the baryon case [17]. The qq potential reads (there is a factor of difference 1/2 with respect to the $q\bar{q}$ case, coming from the assumed $\vec{\lambda}_i \cdot \vec{\lambda}_j$ color structure):

$$V(r_{ij}) = \frac{1}{2} \left[-\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a^2} + \frac{\hbar^2 \kappa_\sigma}{m_i m_j c^2} \frac{e^{-r_{ij}/r_0}}{r_0^2 r_{ij}} (\vec{\sigma}_i \cdot \vec{\sigma}_j) \right] \quad (1)$$

where r_{ij} is the interquark distance, m_i the mass of quark i , $\vec{\sigma}$ the Pauli matrices and κ , a and κ_σ are constants. The hyperfine (spin-spin) interaction has been regularized in order to avoid the unbound spectrum that would cause a contact term, $\delta(\vec{r}_{ij})$, as obtained from the nonrelativistic reduction of the OGE diagram in QCD. The values of the parameters appear in Table I.

Fig. 1 shows a part [for $N(1/2^+)$ and $\Delta(3/2^+)$] of the predicted nonstrange baryon spectrum and its comparison to data. We have plotted by the solid line the $L = 0$ multiplets (L is the total orbital angular momentum) and by the dashed line the first two excited states with $L \neq 0$. As clearly seen two major problems show up. On the one hand the position of Roper resonances (first positive parity excitations) for N and Δ , on the other hand the proliferation of baryon states above 1 GeV excitation energy that have not been experimentally seen. The first problem, say the location in energy of the Roper resonances, has two different aspects, one is the high excitation energy predicted that comes in part from the large strength used for the Coulomb interaction, the other is its inverted position (with respect to data) relative to the first negative parity states. This problem was long-ago related to relativistic corrections [19]. Since then many different solutions have been proposed [3,5,20–25]. For the second problem, say the theoretically predicted excess of high energy excitations (missing state problem), the best known proposed solution is based on the weak pion coupling that most of these predicted excited states may have what would make them very difficult to be detected experimentally [6,7,26–28].

III. SCREENED POTENTIAL

The consideration in the lattice of sea quarks apart from valence quarks (unquenched approximation) suggests a screening effect on the potential when increasing the interquark distance [1]. Creation of light $q\bar{q}$ pairs out of vacuum in between the quarks becomes energetically preferable resulting in a complete screening of quark color charges at large distances (in an adiabatic approach this could be interpreted as if the confining energy ceases to increase because it is also employed in the pair creation). Then the breaking of color flux tubes, or the splitting of the quarks, may occur. Beyond the splitting energy the same interaction with the sea quarks will give rise to hadronization. Otherwise said the decay process takes place.

In the 80's a specific parametrization of these effects was given in the form of a screening multiplicative factor in the potential reading $[(1 - e^{-\mu r_{ij}}) / \mu r_{ij}]$ where μ is a screening parameter [10]. As a consequence the hadron spectrum becomes finite. Let us realize that for sufficiently larger distances the screened linear term in the potential becomes the dominant one. Thus the highest excited states are presumably "confinement states", i. e. they may be determined by considering only the screened linear interaction. The other way around, from the experimentally detected higher excited states one might infer approximate values of the screened linear potential parameters. Unfortunately for the heavy meson spectrum high energy excitation data are very poor and we cannot pursue the phenomenological analysis in this manner. On the contrary, if we assume the same form of screening for light quarks, the nonstrange baryon spectrum seems to be promising to this respect. For the sake of simplicity we will restrict ourselves to a screened Bhaduri-type potential. Following the notation employed in Eq. (1), we write the potential:

$$V_b(r_{ij}) = \frac{1}{2} \left[\frac{r_{ij}}{\bar{a}^2} - \frac{\bar{\kappa}}{r_{ij}} + \frac{\hat{\hbar}^2 \hat{\kappa}_\sigma}{m_i m_j c^2} \frac{e^{-r_{ij}/\bar{r}_0}}{\bar{r}_0^2 r_{ij}} (\vec{\sigma}_i \cdot \vec{\sigma}_j) \right] \left(\frac{1 - e^{-\mu_b r_{ij}}}{\mu_b r_{ij}} \right) \quad (2)$$

where the hat over the parameters indicates that they are different than in the nonscreened case.

Notice that there will be a splitting energy, equal or bigger than $3/(2\bar{a}^2 \mu_b)$ (since there are three pairs of quarks), for the $3q$ system to be split into three free quarks. But equally important in this case in order to analyze the stability of the system is the calculation of the energy thresholds for only one quark to be released. For $3q$ binding energies higher than these thresholds a ($2q$ state + $1q$ free state) will be a more probable configuration that, through hadronization, will decay into a baryon and a meson or multimeson states. From now on we shall refer to these energy thresholds as $1q$ ionization thresholds. Let us note that as these energy thresholds may be well below the top of the $3q$ effective potential, they do not completely prevent the possible existence of metastable $3q$ bound states at higher energy assumed that for dynamical reasons the induced decays are suppressed.

Concerning the value of the parameters we can give first some arguments of plausibility before going to a more refined fit from data. As the screening multiplicative factor reduces to 1 when $r_{ij} \rightarrow 0$ the potential comes dominated in this limit by the OGE piece with exactly the same form than in the nonscreened case. So we expect the parameters of the OGE interaction, $\bar{\kappa}$, $\hat{\kappa}_\sigma$ and \bar{r}_0 to be quite close to the corresponding values of κ , κ_σ and r_0 given

in Sec. II in order to get a similar description of the low-lying baryon spectrum. However as commented above the large Coulomb strength in the nonscreened case is to a good extent responsible for the incorrect position predicted for the radial excitations. So we shall reduce $\bar{\kappa}$ against κ as much as the preservation of the quality of the predicted spectrum allows. This has the bonus of reducing the relative importance of the Coulomb interaction versus the linear one what is very convenient for a fit of the screened linear potential parameters from the highest energy excitation data.

With respect to $1/(2\bar{a}^2)$ we expect its value to be bigger than $1/(2a^2) = 470.5 \text{ MeV fm}^{-1}$ from Sec. II. In this manner the linear potential strength reduction coming from the screening can be compensated and the quality of the medium-lying baryon spectrum obtained in Ref. [17], for which the linear term was already playing some role, not spoiled. Regarding μ_b we expect it to be, in the light quark case, a completely effective parameter.

In order to be more precise we center our attention in the experimental higher energy resonances: there are a $N(7/2^-)$ state at 2190 MeV, a $N(9/2^+)$ at 2220 MeV, a $N(9/2^-)$ at 2250 MeV and a $\Delta(11/2^+)$ at 2420 MeV cataloged as well established (***) states by the Particle Data Group [18]. For the nucleon there is also a very likely $N(11/2^-)$ (**) state at 2600 MeV and a fair evidence of a $N(13/2^+)$ (**) at 2700 MeV. For the Δ there are two more uncertain states, the $\Delta(13/2^-)$ (**) at 2750 MeV and the $\Delta(15/2^+)$ (**) at 2950 MeV. It turns out very difficult to reasonably accommodate within the experimental errors (also more uncertain for resonances above 2500 MeV) and with the same screened linear potential the well established resonances and the uncertain ones altogether. Assuming that this may be an indication of the presence of the continuum we shall restrict ourselves to the highest (***) particles, that we shall consider to be close to threshold, to fix the screened linear potential parameters.

IV. RESULTS

To get the nonstrange baryon spectrum we proceed to solve the Schrödinger equation by two different procedures: i) the hyperspherical harmonic (HA) expansion method [29] and ii) the Faddeev method. The HA treatment allows a more intuitive understanding of the wave functions in terms of the hyperradius of the whole system. As a counterpart one has to go to a very high order in the expansion to get convergence. To assure this we shall expand up to $K = 24$ (K being the great orbital determining the order of the expansion). In the Faddeev calculation in order to assure convergence we shall include (l, λ, s, t) configurations (l is the orbital angular momentum of a $2q$ pair, λ is the orbital angular momentum of the third quark with respect to the center of mass of the $2q$, and s and t are the spin and isospin of the $2q$ respectively) up to $l=5$ and $\lambda=5$ [4]. Differences in the results for the $3q$ bound state energies obtained by means of the two methods turn out to be at most of 5 MeV. Let us realize that we can calculate with precision with any method only below or close above (by a continuity procedure) the $1q$ ionization thresholds. Higher in energy the oscillatory behavior corresponding to the $1q$ free state introduces numerical uncertainties and the level of convergence is lost.

The predicted spectra for N and Δ are presented in Figs. 2 and 3 and compared to data. The values of the parameters obtained after some fine tuning to better accommodate the

data appear in Table II. The $1q$ ionization thresholds for the different values of (l, s, t) are shown in Table III and plotted in Figs. 2 and 3 for the different J^P states.

Let us realize first that at low and medium excitation energies the spectrum obtained is, according to our hope, of similar or even better quality (due to the smaller Coulomb strength) than the corresponding to the nonscreened Bhaduri potential. Regarding the higher excitations the quality of the description is remarkable (let us note that we have also included the states we predict close above the thresholds since we do not pretend to have a precise dynamical description of the thresholds with our simple model) since apart from keeping the same level of quality than in the low and medium-lying spectrum we get a perfect (one to one) correspondence between our predicted states and the experimental (up to 2500 MeV) resonances for any J^P . The only *exception* appears in the $\Delta(3/2^-)$ for which we predict three states and there are two cataloged resonances by the Particle Data Group. However in our opinion it seems reasonable that the quite different masses reported in different measurements for the highest state (1940 and 2057 MeV) may correspond in fact to two different states. As a consequence if we assume that the quantitative differences we have with data are due to the lack of a more complete dynamical treatment that does not change our qualitative result, the missing resonance problem, as stated in Sec. II, disappears, i. e. there are no missing resonances that correspond to quark model $3q$ bound states. The price to be paid is the appearance of free quark states what requires a hadronization mechanism to connect them to phenomenology. We assume that the same pair creation mechanism responsible for screening induces the hadronization of a $(2q$ state $+1q$ free state) into a baryon-meson state.

With respect to the values of the parameters our initial expectancies are confirmed after the fine tuning process to fit the spectrum. For the spin-spin interaction we have very similar values to the nonscreened case and for $1/(2\bar{a}^2)$ a quite bigger value as can be checked from Table I. Finally for the Coulomb strength we are able to reduce its value to approximately one tenth of the nonscreened case value, what causes an improvement in the description of the radial excitations. This supposes a big difference between the Coulomb and spin-spin strengths (they are equal in the non-screened case) indicating the very effective character of the parameters. It is worthwhile to mention that if instead we had maintained the nonscreened Coulomb strength we would have had the possibility to include in our description some of the highest uncertain states but at the price of losing quality in the overall fit to the spectrum.

For our choice of the parameters no $3q$ bound state configuration exists with a minimum value of $l + \lambda > 4$. We can understand this result by drawing in Fig. 4 an average effective baryonic potential defined as three times the quark-quark potential plus a centrifugal barrier specified by orbital angular momentum l or λ . This is

$$V_{eff}(r_{ij}) = 3V_b(r_{ij}) + \frac{\hbar^2 l(l+1)}{2 m_{red} r_{ij}^2} \quad (3)$$

where m_{red} is the reduced mass of a pair of quarks. For this potential the splitting energy is $3/(2\bar{a}^2 \mu_b)$ and no bound state can be accommodated for $l \geq 5$. For $l \leq 4$, apart from the possibility of having bound states below the $1q$ ionization thresholds, there could also exist, from the $1q$ ionization thresholds up to the splitting energy, some metastable $3q$ states as

explained above. In our case the nonconsidered resonances above 2500 MeV could be of this type provided that their uncertain experimental errors leave opened the possibility for them to be below the total splitting energy of the system.

It may also be interesting to draw the wave functions. In Fig. 5(a) we plot the ground state nucleon wave functions for the screened and nonscreened Bhaduri potentials. As we can check the presence of screening does not mean any major effect but a slight enhancement probability at short distances compensated by a slight depression at medium range. Just for comparison we show in Fig. 5(b) the wave functions for the ground state of the dominant component of the $N(5/2^+)$ where we can appreciate a more significant difference due to the fact that the large distance part of the potential is playing a more relevant role.

V. SUMMARY

We have examined the consequences, for N and Δ spectra, of a quark model description based on a screened potential. The form chosen for the potential is quite simple containing a linear term plus the Coulomb and spin-spin terms of the usual OGE interaction, both screened through a Yukawa type factor. The OGE parameters, strength for the Coulomb and strength and range for the spin-spin terms, are fixed from the low-lying part of the spectrum whereas the screened linear potential parameters, strength and range (screening), are determined from the higher experimental excitations. The resulting average quark velocities inside the baryon, close to c (one would obtain even bigger values if the nonrelativistic expression, p_q/m_q , were applied) make clear the shortcomings of the non-relativistic quark model treatment. Nevertheless the description of the spectrum is quite satisfactory. Except for a few cases [$N(1440)$, $\Delta(1910)$] our results differ at most 100 MeV from data for well established (***) and (***) resonances. Concerning the non-well established resonances it is also remarkable that below 2500 MeV most of our predictions lie inside the current experimental error bars. The failure in the description of particles beyond 2500 MeV may be either an indication of the limit of applicability of the model, or an indication that we are in the transition to the continuum including the possibility of metastable states, or a signature of the presence of exotics (non $3q$ bound states). Anyhow an experimental effort to clarify the situation in this energy region seems to be mandatory. Keeping this in mind we can say that the major distinctive consequence of the use of a screened potential is the finiteness of the spectrum resulting in an perfect parallelism between predicted $3q$ bound states and experimental resonances. As a counterpart to this striking result we are left with a non-confining model. A dynamical understanding of the decay mechanism to $1q$ free states, involving hadronization, beyond our qualitative reasonings is mandatory. When this program is carried out it may well merge out a different view on the so-called missing resonance problem as an indication that confinement has to be properly implemented. In this sense we should say that we do not expect our screening parametrization and our dynamical thresholds for the existence of $3q$ bound states to be a precise description of the underlying QCD dynamics (the cross-over from a linear to a flat potential can be much more rapid in QCD). Despite this the good quality of the high energy spectrum obtained and its perfect correspondence to data drive us to think that the qualitative consequences of screening we have derived will be maintained for more refined potentials. To pursue the effort along this

line, beyond our exploratory work, could help in our opinion to a better knowledge of the essential ingredients to get an adequate description of hadrons.

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TABLES

TABLE I. Quark model parameters used in Ref. [17].

$m_u = m_d$ (MeV)	337
r_0 (fm)	0.45
κ (MeV fm)	102.67
κ_σ (MeV fm)	102.67
a [(MeV) $^{-1}$ fm] $^{1/2}$	0.0326

TABLE II. Quark model parameters for the calculation of Figs. 2 and 3.

$m_u = m_d$ (MeV)	337
\bar{r}_0 (fm)	0.477
$\bar{\kappa}$ (MeV fm)	10.0
$\bar{\kappa}_\sigma$ (MeV fm)	120.0
\bar{a} [(MeV) $^{-1}$ fm] $^{1/2}$	0.0184
μ_b (fm $^{-1}$)	1.05

TABLE III. $1q$ ionization thresholds.

(l, s, t)	E(MeV)
(0, 0, 0)	1007
(0, 1, 1)	1156
(1, 0, 1)	1388
(1, 1, 0)	1402

FIGURES

FIG. 1. Relative energy $N(1/2^+)$ and $\Delta(3/2^+)$ spectrum up to 2.5 GeV excitation energy for the potential of Ref. [17]. The solid and dashed lines correspond to the Faddeev results including ℓ and λ up to 5, and for $L < 3$ as explained in the text. The shaded regions, whose size stands for the experimental uncertainty, represent three or four star resonances in the notation of Ref. [18]. The energy of the one star resonance in the $N(1/2^+)$ case is denoted by a thin solid line with one star over it and its experimental uncertainty by a vertical line with arrows.

FIG. 2. Relative energy nucleon spectrum for the screened potential. The solid lines represent our results. The shaded region, whose size stands for the experimental uncertainty, represents the experimental data for those states cataloged as (***) or (****) states in the Particle Data Book [18]. Experimental data cataloged as (*) or (**) states are shown by solid lines with stars over them and by vertical lines with arrows standing for the experimental uncertainties. Finally, we show by a dashed line the $1q$ ionization threshold and by a thin solid line the total threshold.

FIG. 3. Same as Fig. 2 for Δ states.

FIG. 4. Effective potential as written in Eq. (3) for different values of l , the reduced mass of the light quarks and spin one.

FIG. 5. (a) Radial wave function of the completely symmetric component of the $N(1/2^+)$ in terms of the hyperradius ρ . The dotted line stands for the result of Ref. [17], the solid line corresponds to the screened potential. (b) Same as (a) for the dominant component of the $N(5/2^+)$ ground state.

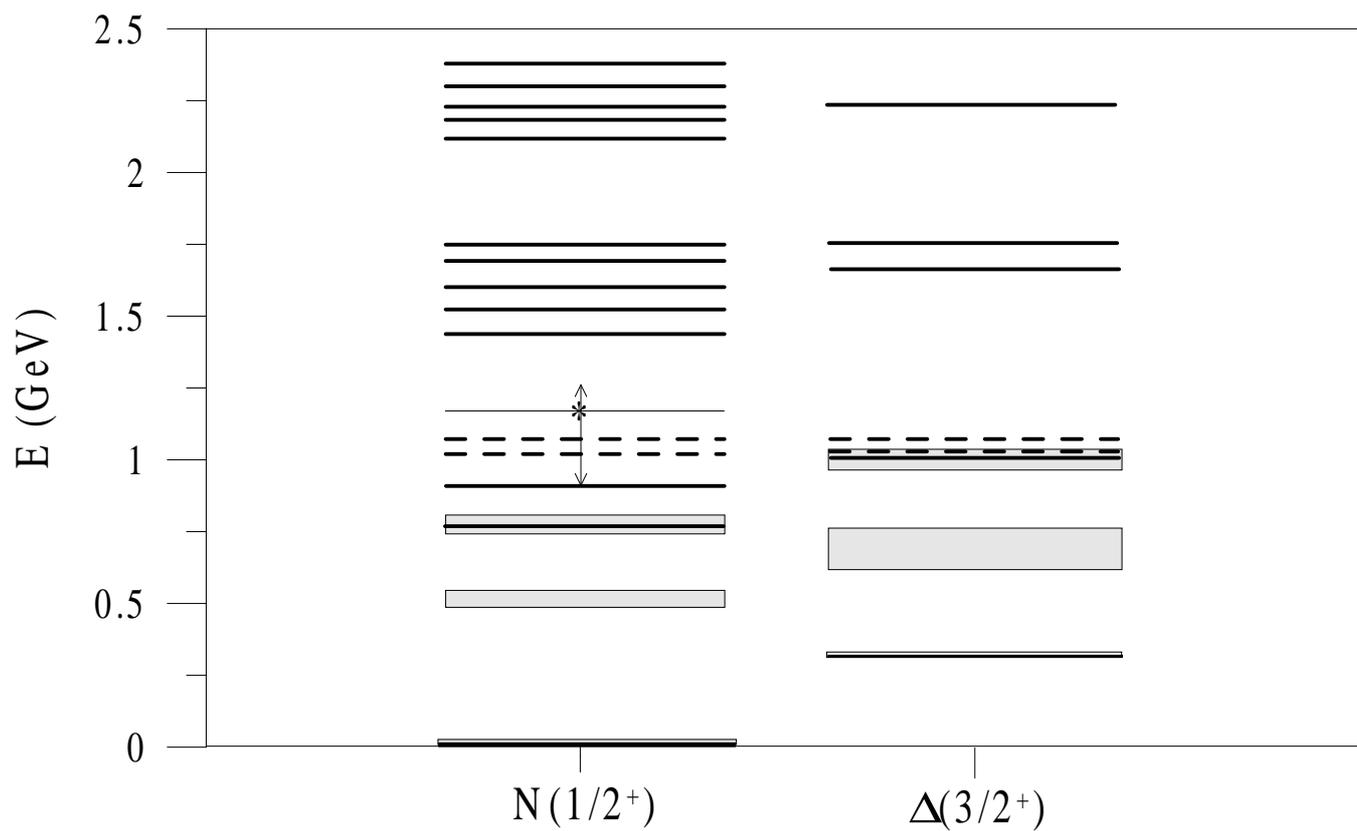


Figure 1

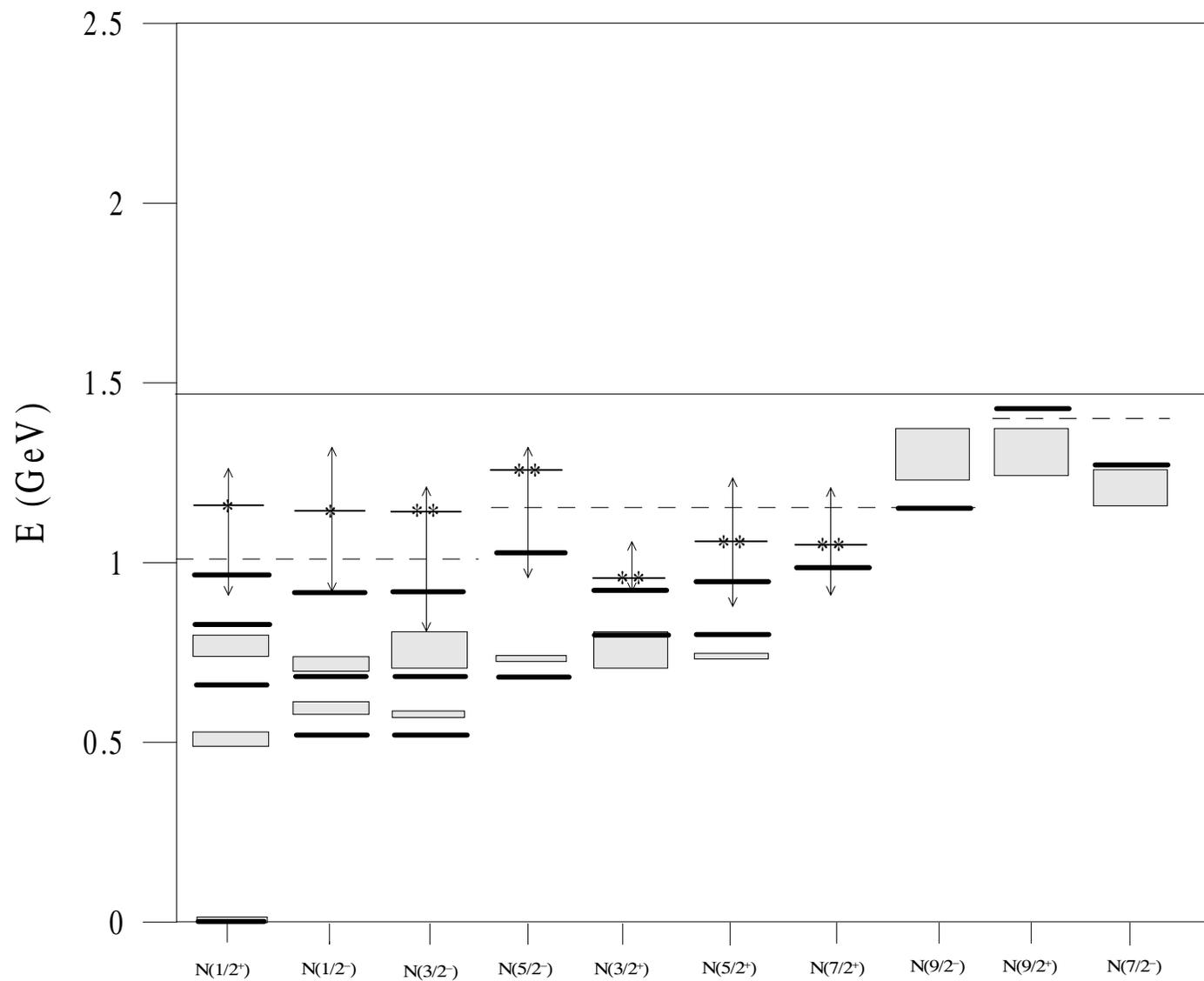


Figure 2

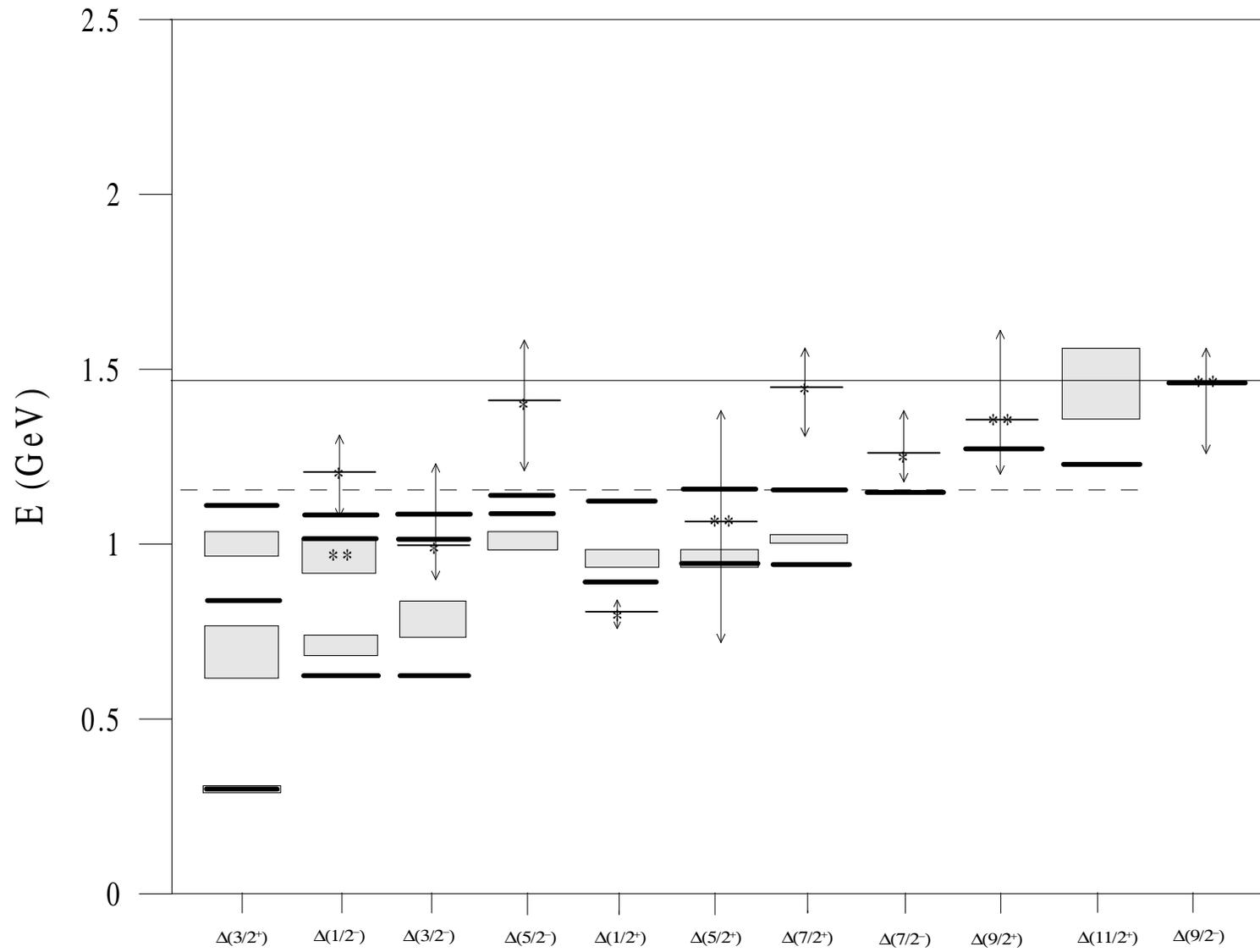


Figure 3

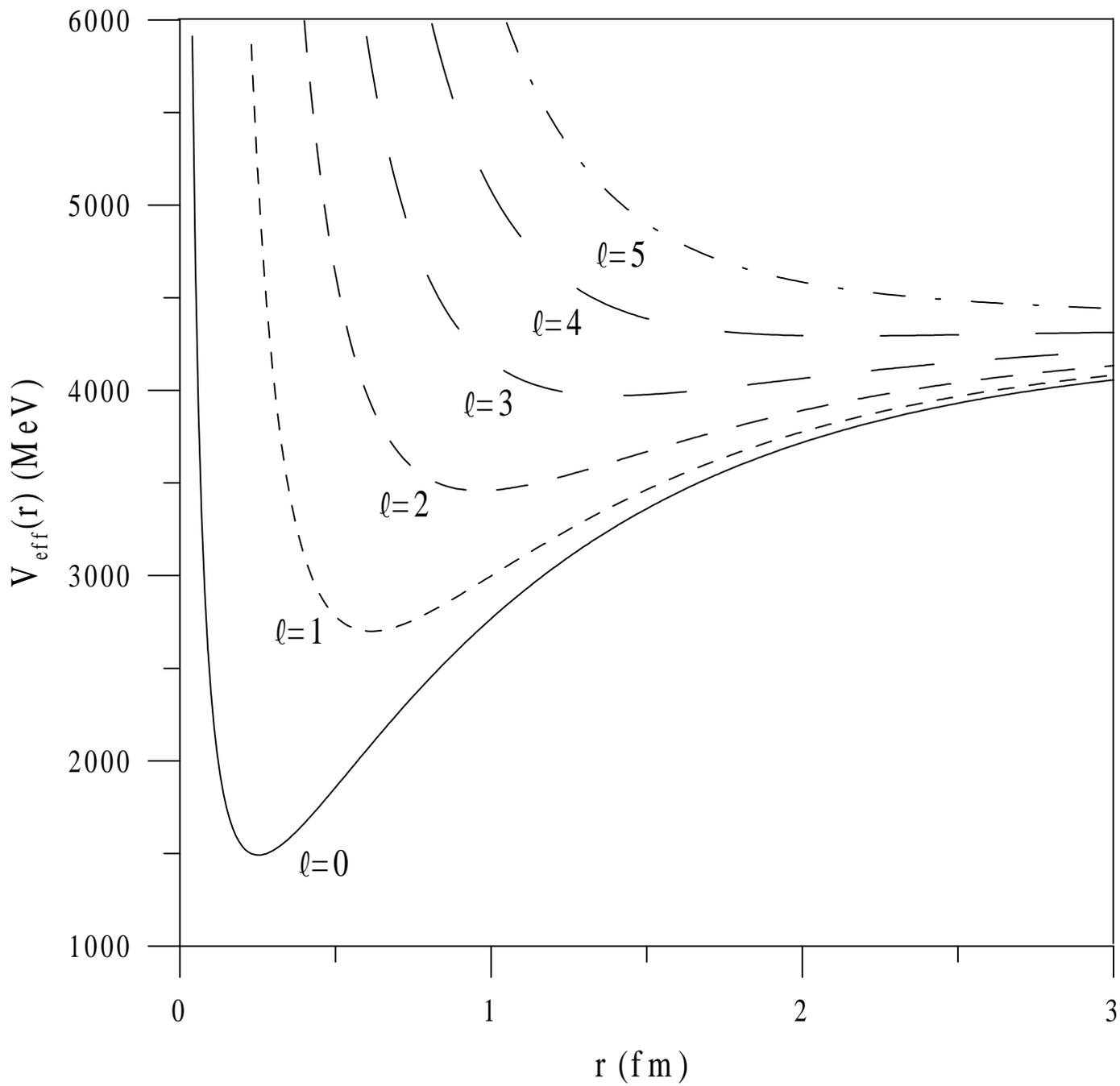


Figure 4

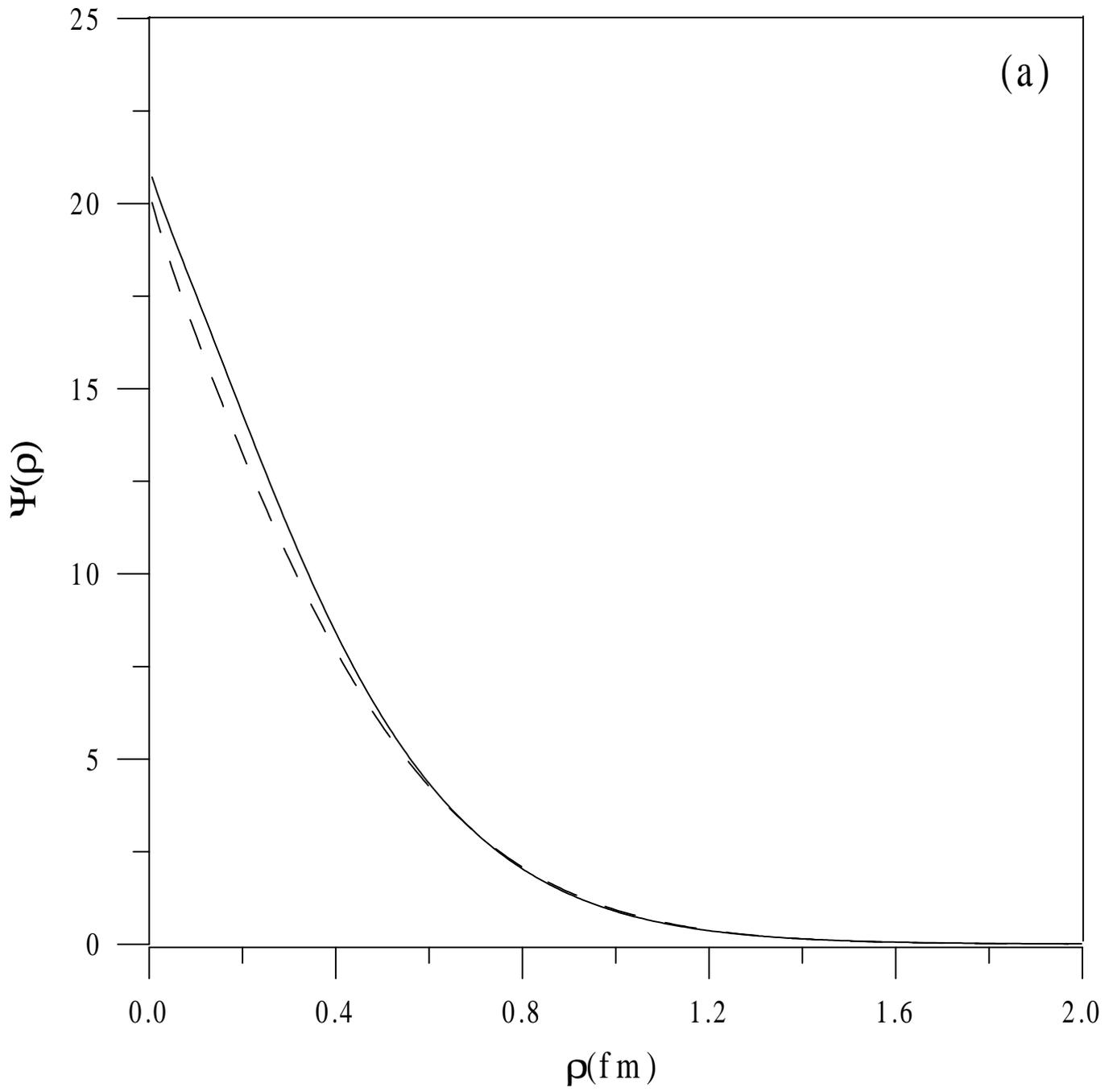


Figure 5

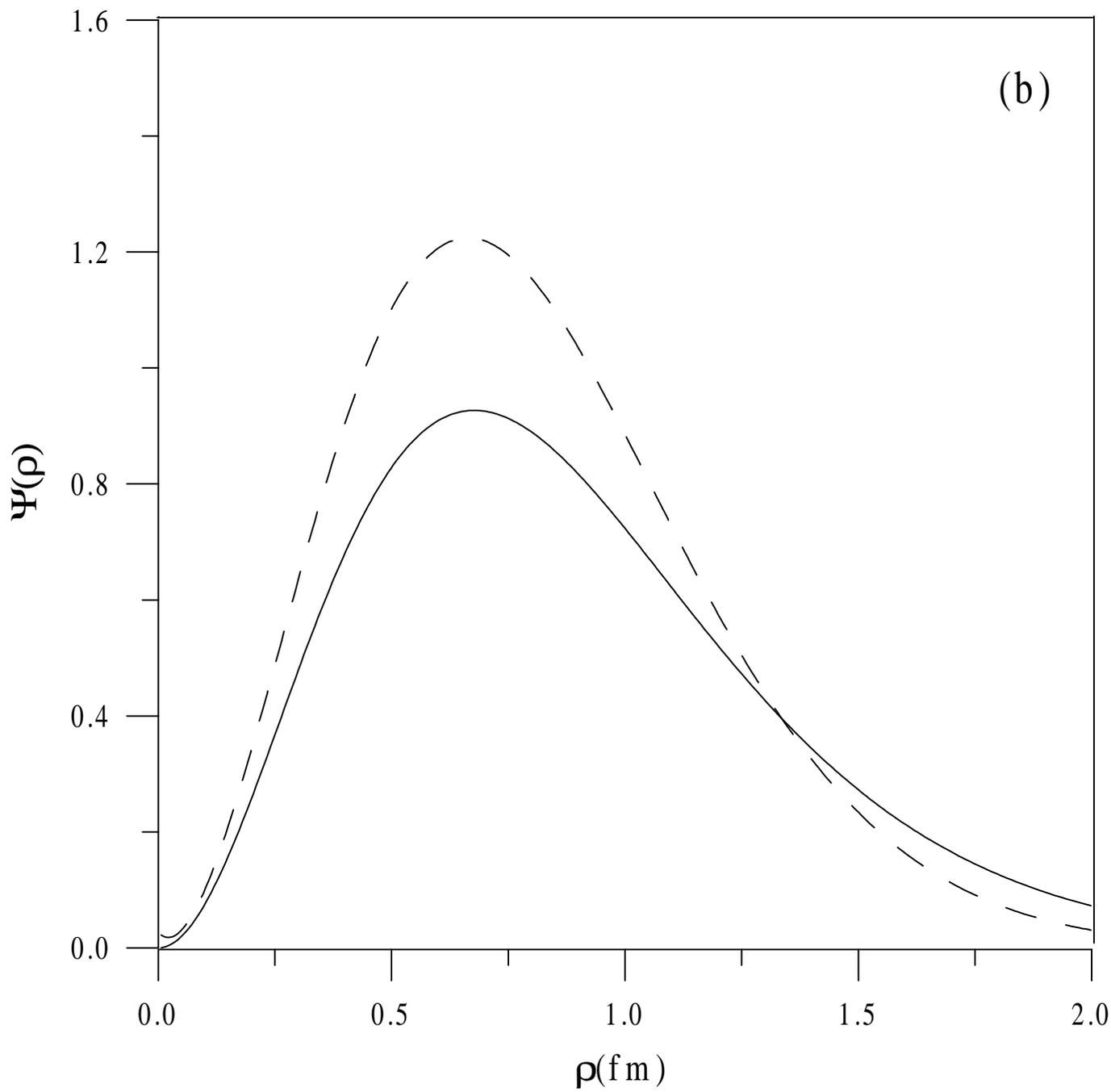


Figure 5