Predicting Neutrinoless Double Beta Decay

M. Hirsch† A. Villanova del Moral§ and J. W. F. Valle∗
AHEP Group, Instituto de Física Corpuscular – C.S.I.C./Universitat de València
Edificio Institutos de Paterna, Apt 22085, E-46071 Valencia, Spain

Ernest Ma†
Physics Department, University of California, Riverside, CA 92521, USA, and
Institute for Particle Physics Phenomenology, University of Durham, Durham, DH1 3LE, UK
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We give predictions for the neutrinoless double beta decay rate in a simple variant of the $A_4$ family symmetry model. We show that there is a lower bound for the $ββ_0$ amplitude even in the case of normal hierarchical neutrino masses, corresponding to an effective mass parameter $|m_{ee}| ≥ 0.17\sqrt{\Delta m_{32}^2}$. This result holds both for the CP conserving and CP violating cases. In the latter case we show explicitly that the lower bound on $|m_{ee}|$ is sensitive to the value of the Majorana phase. We conclude therefore that in our scheme, $ββ_0$ may be accessible to the next generation of high sensitivity experiments.

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Are neutrinos their own antiparticles? This is one of the most basic current unknowns in neutrino physics. Of all possible manifestations of neutrino masses, neutrinoless double beta decay ($ββ_0$, for short) offers – to date – the only potentially viable way to answer this question. If $ββ_0$ exists, then neutrino masses are Majorana in nature, irrespective of their ultimate origin [1]. Together with cosmology [2] and direct kinematical searches in tritium decay [3], $ββ_0$ offers one of the three main complementary ways to probe the absolute scale of neutrino masses.

Current experimental limits from $ββ_0$ on the effective Majorana mass of the neutrino $m_{ee}$ are of order $m_{ee} ≤ 0.3 – 1$ eV [4, 5]. We note that a claim for a finite $ββ_0$ rate has been published in [6], but this has so far not been confirmed by any other experiment. A recent proposal [7] aims explicitly at testing the half-life range suggested in [6]. However, future $ββ_0$ experiments may be able to reach down to much lower mass scales. Many experiments sensitive to $m_{ee} ≃ 0.05$ eV have already been discussed; see for example [8, 9]. In the longer-term future, even $m_{ee} ≃ 0.01$ eV [10, 11] does not seem impossible.

The historic confirmation of neutrino oscillations over the last few years [12], together with some basic theory, suggests that $ββ_0$ is expected, although in general no lower bound on the magnitude of the expected effect can be given. Theoretical input is therefore needed. Currently the origin of neutrino masses is completely unknown. The basic dimension–five operator which leads to neutrino masses [13] can arise from a variety of mechanisms characterized by vastly different scales. Alternatives include the seagull mechanism [14, 15, 16] and low–energy $R$–parity violating supersymmetry [17]. In neither case is it possible, in general, to establish a lower bound on the magnitude of the $ββ_0$ rate.

Here we consider a simple phenomenological model based on a new realization of the $A_4$ family symmetry [18, 19, 20] in which no SU(3) ⊗ SU(2) ⊗ U(1) singlet neutrinos are introduced. Instead, the small neutrino masses arise from the small induced vacuum expectation values (VEVs) generated for the neutral components of triplet Higgs bosons [12, 21], transforming nontrivially under the $A_4$ family symmetry. The lepton and Higgs particle content and their transformation properties under $A_4$ and SU(2) ⊗ U(1) are specified in Table I. With

$$M_β = \begin{pmatrix}
  a + b + c & f & e \\
  f & a + \omega b + \omega^2 c & d \\
  e & d & a + \omega^2 b + \omega c
\end{pmatrix}$$

(1)

### Table I: Lepton and scalar boson quantum numbers

<table>
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<tr>
<th>Fields</th>
<th>L</th>
<th>f</th>
<th>φ1</th>
<th>φ2</th>
<th>φ3</th>
<th>η1</th>
<th>η2</th>
<th>η3</th>
<th>ξ</th>
</tr>
</thead>
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<td>3</td>
<td>1′</td>
<td>1′</td>
<td>1′</td>
<td>1′</td>
<td>1′</td>
<td>1′</td>
<td>3</td>
</tr>
<tr>
<td>SU(2)</td>
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<td>1</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Y</td>
<td>−1</td>
<td>2</td>
<td>−1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

These transformation properties, the charged lepton mass matrix is already diagonal in the flavor basis, with

$$m_e = h_1 v_1 + h_2 v_2 + h_3 v_3$$
$$m_μ = h_1 v_1 + \omega h_2 v_2 + \omega^2 h_3 v_3$$
$$m_τ = h_1 v_1 + \omega^2 h_2 v_2 + \omega h_3 v_3$$

where $h_i$ are charged lepton Yukawa couplings, $v_i = \langle φ_i^0 \rangle$ and $ω$ is a complex cubic root of unity satisfying $1 + ω + ω^2 = 0$. The neutrino mass matrix is then of the form

$$M_ν = \begin{pmatrix}
  a + b + c & f & e \\
  f & a + \omega b + \omega^2 c & d \\
  e & d & a + \omega^2 b + \omega c
\end{pmatrix}.$$

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*Electronic address: Martin.Hirsch@ific.uv.es
†Electronic address: Albert.Villanova@ific.uv.es
‡Electronic address: valle@ific.uv.es
§Electronic address: ma@physun8.ucr.edu
where the only non-diagonal entries are those of the $A_4$ triplet $\xi$. Here we have defined
\[
\begin{align*}
a &= \lambda_1 \langle \eta_1^0 \rangle \\
b &= \lambda_2 \langle \eta_2^0 \rangle \\
c &= \lambda_3 \langle \eta_3^0 \rangle \\
d &= \kappa \langle \xi_1^0 \rangle \\
e &= \kappa \langle \xi_2^0 \rangle \\
f &= \kappa \langle \xi_3^0 \rangle
\end{align*}
\]
where $\lambda_i$, $\kappa$ and $\langle \eta_i^0 \rangle$, $\langle \xi_i^0 \rangle$ are triplet Yukawa couplings, and VEVs, respectively. Let us further assume that the conditions $b = c$, and $d = e = f$ hold. Whereas the former is an ad hoc assumption, the latter can be maintained naturally because of a residual $Z_3$ symmetry. Note that $M_\nu$ has a very remarkable (and possibly unique) property here in that each entry is renormalized by the charged-lepton Yukawa couplings possibly unique) property here in that each entry is renormalized by the charged-lepton Yukawa couplings.

It is easy to see that in the above limit we have the prediction
\[
\theta_{23} = \pi/4, \quad \theta_{13} = 0
\]
which matches well with the neutrino oscillation data [12]. Furthermore it can be shown that, in the limit where the solar mass splitting is neglected, $b$ and $d$ can be made real, so that the atmospheric neutrino mass splitting takes on a very simple form
\[
\Delta m^2_{32} = 6bd \equiv \Delta m^2_{\text{ATM}}
\]
The solar neutrino mass splitting $\Delta m^2_{\text{sol}} \ll \Delta m^2_{\text{ATM}}$ can be expressed as
\[
\Delta m^2_{21} = \sqrt{T_1^2 + T_2^2 + T_3^2} \equiv \Delta m^2_{\text{sol}}
\]
where
\[
\begin{align*}
T_1 &= 6\sqrt{2}|b||d| \sin(\phi_2) \\
T_2 &= 2\sqrt{2}|d| \left(2|a| \cos(\phi_1) + |b| \cos(\phi_2) + |d| \right) \\
T_3 &= -3|b|^2 + |d|^2 - 6|a||b| \cos(\phi_1 + \phi_2) + 2|a||d| \cos(\phi_1) - 2|b||d| \cos(\phi_2)
\end{align*}
\]
with $\phi_1 \equiv \phi_a - \phi_d$ and $\phi_2 \equiv \phi_d - \phi_b$, where $a = |a| \exp(i\phi_a)$, $b = |b| \exp(i\phi_b)$ and $d = |d| \exp(i\phi_d)$. The condition in Eq. (4) leads to three inequalities
\[
|T_i| \leq \Delta m^2_{\text{sol}}
\]
which, normalized by $\Delta m^2_{\text{ATM}}$, can be expressed in terms of the small parameter $\alpha \equiv \Delta m^2_{\text{sol}}/\Delta m^2_{\text{ATM}}$ as
\[
\sqrt{2} \sin(\phi_2) \leq \alpha
\]
and
\[
\frac{\sqrt{2}}{3|b|} \left|2|a| \cos(\phi_1) + |b| \cos(\phi_2) + |d| \right| \leq \alpha
\]
where the current allowed values of $\alpha$ are shown in Fig. 13 of Ref. [12].

As for the solar mixing angle, it is given here by
\[
t_{2S} \equiv \tan(2\theta_{12}) = \frac{2\sqrt{2d}}{3b - d}
\]
which reduces to
\[
\tan^2 \theta_{12} = 1/2
\]
in two ways, namely,
\[
b = 0, \quad b = 2d/3
\]
Current fits of solar, reactor, atmospheric and accelerator neutrino oscillation data lead to a best fit point to the solar mixing angle for which $\tan^2 \theta_{12}$ is slightly less than $1/2$. Performing a series expansion of $\tan^2 \theta_{12}$ around the two solutions in Eq. (13), we get
\[
\tan^2 \theta_{12} \simeq \frac{1}{2} + \frac{b}{d}
\]
\[
\tan^2 \theta_{12} \simeq \frac{1}{2} - \frac{1}{d \left(b - \frac{2}{3}d\right)}
\]
These two branches are depicted in Fig. 1. The positive horizontal axis corresponds to a normal hierarchy (NH) neutrino spectrum, while the negative part corresponds to inverse hierarchy (IH), as depicted in Fig. 1. This behavior is also recognized in the model of [22].

We now turn to neutrinoless double beta decay. The neutrino–exchange amplitude for this process is simply given by
\[
\langle m_\nu \rangle = m_{ee} = a + 2b
\]
In the case of real parameters, we can solve the following
system of equations
\[
\begin{align*}
\Delta m_{\text{sol}}^2 &\equiv \Delta m_{21}^2 = |2a + b + d| \sqrt{(d - 3b)^2 + 8d^2} \\
\Delta m_{\text{ATM}}^2 &\equiv \Delta m_{32}^2 = 6bd \\
t_{2S} &\equiv \tan(2\theta_{12}) = \frac{2\sqrt{3d}}{3b - d}
\end{align*}
\]
and express the parameters \(a\), \(b\) and \(d\) in terms of experimentally measurable ones \(\Delta m_{\text{sol}}^2\), \(\Delta m_{\text{ATM}}^2\) and \(t_{2S}\). Substituting in Eq. (16) we can therefore express \(\langle m_\nu \rangle\) in terms of these measured observables. We then obtain, up to an overall sign,
\[
\frac{m_{ee}}{\sqrt{\Delta m_{\text{ATM}}^2}} = \text{Sign}[\Delta m_{\text{ATM}}^2] \frac{1}{\sqrt{2\sqrt{2}t_{2S} + t_{2S}^2}} \pm \text{Sign}[2\sqrt{2}t_{2S} + t_{2S}^2] \frac{\alpha \sqrt{2\sqrt{2}t_{2S} + t_{2S}^2}}{4\sqrt{1 + t_{2S}^2}}
\]
The calculated values of \(|m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}|\) as functions of \(t_{2S}\) according to Eq. (17) are shown in Fig. 2. It can be seen that given the currently allowed experimental 3\(\sigma\) range
\[
\tan^2 \theta_{12} \in [0.30, 0.61]
\]
we can set lower bounds for \(|m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}|\):
\[
\begin{align*}
0.23 < |m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}| &< 0.41 \quad \text{for NH} \quad (19) \\
0.70 < |m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}| &< 0.84 \quad \text{for IH} \quad (20)
\end{align*}
\]
Note that the lower bound on \(\beta\beta_{0v}\) for the NH case is especially relevant, as it is absent in the generic case. It results from this specific realization of the \(A_2\) symmetry and is also in contrast with previous \(A_4\)-based models that led to quasi-degenerate neutrinos\[15, 23\].

Moreover, should future precision experiments narrow down the experimental range for \(\tan^2 \theta_{12}\) then we might be able to distinguish between both neutrino mass hierarchies. For example, if \(\tan^2 \theta_{12} \leq 1/2\) could ever be established, then we would have
\[
\begin{align*}
0.23 < |m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}| &< 0.41 \quad \text{for NH} \quad (21) \\
0.70 < |m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}| &< 0.84 \quad \text{for IH} \quad (22)
\end{align*}
\]
It can also be seen that, up to order \(\alpha\) corrections, \(m_{ee}\) can only be zero if \(\tan^2 \theta_{12} = 1\), now strongly rejected experimentally. Note that the two solutions in Fig. 2 correspond to the two branches depicted in Fig. 1. One can see that only in the branch corresponding to the solution \(b = 0\) and for values of \(\tan^2 \theta_{12}\) that are less than \(1/2\), there is a relative minus sign between \(b\) and \(d\), which is the condition for inverse hierarchy, as can be seen from Eq. (16).

In the general case of complex parameters, lower bounds on \(|m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}|\) can also be established for each hierarchy. This task is simplified greatly by taking into account the reliable approximations \(\sin \phi_2 \approx 0\), and \(2|a|\cos \phi_1 + |b|\cos \phi_2 + |d| \approx 0\). From Fig. 3 we see that in the complex case, lower bounds on \(|m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}|\) are indeed also established for each hierarchy. By comparing Fig. 3 with the right panel in Fig. 2 we find that these lower bounds are in fact exactly the same as obtained in the real case. The robustness of these bounds is easily understood. It follows from the fact that the maximum degree of destructive interference between the three neutrino–exchange contributions occurs in the real case with appropriate CP parities\[24\].

![FIG. 2: \(|m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}|\) vs. \(t_{2S}\) according to Eq. (17). The bound depends slightly on the value of \(\alpha\): the left panel corresponds to \(\alpha = 0.022\) and the right one to \(\alpha = 0.065\). The dark (red) lines correspond to normal hierarchy, while the grey (green) line is for the inverse hierarchy case. The vertical line corresponds to the horizontal line in Fig. 1 at \(\tan^2 \theta_{12} = 1/2\).](image1)

![FIG. 3: \(|m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}|\) vs. \(t_{2S}\) for \(\alpha = 0.065\) when complex parameters are allowed.](image2)
Last, but not least, we find it very interesting also that the lower bound on $m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}$ which we have obtained depends on the value of the Majorana violating phase $|\cos(\phi_1)|$. We can see from Fig. 4 that the lower bounds on $|m_{ee}/\sqrt{\Delta m_{\text{ATM}}^2}|$ for each hierarchy become weaker for $\cos(\phi_1) = 1$. Finally, we mention that, even though for simplicity we have focused on the case $b = c$, all results can be generalized to the case $b \neq c$, in which case $\theta_{13}$ is allowed to be nonzero.

In summary, we have given predictions for the neutrinoless double beta decay rate in a simple hierarchical variant of the $A_4$ family symmetry model. We showed that there is a lower bound for the $\beta/\lambda_{\nu}$ amplitude even in the case of normal hierarchical neutrino masses. We have shown that the bound is robust as it holds irrespective of whether CP is conserved or not. In the latter case we show explicitly how the lower bound on $|m_{ee}|$ is sensitive to the value of the Majorana phase. Our scheme suggests that neutrinoless double beta decay may be within reach of the next generation of high sensitivity experiments.

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