High-energy neutrinos may resonate with relic background neutralinos to form short-lived sneutrinos. In some circumstances, the decay chain that leads back to the lightest supersymmetric particle would yield few-GeV gamma rays or charged-particle signals. Although resonant coannihilation would occur at an appreciable rate in our galaxy, the signal in any foreseeable detector is unobservably small.

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The possibility of detecting relic neutrinos by observing the resonant annihilation of extremely energetic neutrinos on the background neutrinos through the reaction $\nu \bar{\nu} \to Z^0$ has been the object of extensive studies $\mathrm{1,2,3}$. Given the small neutrino masses ($\lesssim 1 \text{ eV}$) indicated by current experimental constraints, suitably intense sources of extremely energetic ($10^{21} - 10^{25} \text{ eV}$) cosmic neutrinos are required. The positions and shapes of the absorption lines in the extremely high-energy neutrinos spectrum are influenced by Fermi motion of the relics and by the thermal history of the Universe.

Relic neutrinos have a special standing: According to the standard cosmology, neutrinos should be the most abundant particles in the Universe, after the photons of the cosmic microwave background, provided that they are stable over cosmological times. But the weight of cosmological observations argues that neutrinos are not the only undetected relics. According to the Wilkinson Microwave Anisotropy Probe (WMAP) three-year analysis $\mathrm{4}$, the matter fraction of the present Universe is $\Omega_m h^2 = 0.127^{+0.007}_{-0.014}$, where $h = 0.73 \pm 0.03$ is the reduced Hubble parameter $\mathrm{5}$ and $\Omega_m$ is the ratio of the matter density to the critical density $\rho_c \equiv 3H^2/8\pi G_N$. (Here $H$ is the Hubble parameter and $G_N$ is Newton’s constant.) The baryonic fraction is $\Omega_b h^2 = 0.0223^{+0.0007}_{-0.0009}$, and the neutrino fraction $\Omega_\nu h^2 = (\sum_i m_\nu_i)/94 \text{ eV} \approx 0.0072 \lesssim \Omega_b h^2$. Accordingly, we have reason to believe that the matter fraction is dominated by an unseen “dark-matter” component. A popular hypothesis, realized in many extensions to the standard model including supersymmetry, holds that the dark matter is dominated by a single species of weakly interacting massive particle (WIMP).

There is no confirmed observation of the passage of WIMPs through a detector $\mathrm{6,7}$. A positive signal reported by the dark matter experiment DAMA $\mathrm{8}$ seems in conflict with upper limits set by the Cryogenic Dark Matter Search $\mathrm{9}$ and the EDELWEISS experiment $\mathrm{10}$. Accelerator experiments have so far not yielded evidence for the production of a superpartner candidate for the dark-matter particle $\chi^0$.

In this note, we ask whether resonant coannihilation of neutrinos with dark-matter superpartners might be observable. We study a particular representative case of neutralino dark matter, in which the absolutely stable lightest supersymmetric particle (LSP) is $\chi_1^-$, a superposition of neutralino, wino, bino, and Higgsino. In circumstances apt for detection, resonant formation of a sneutrino in the reaction $\nu \chi_1^0 \to \nu \chi_1^0$, shown in Figure 1, might induce absorption lines or direct signatures. A sneutrino heavier than the neutralino is implied by the assumption that $\chi_1^0$ be the LSP. A direct signature of sneutrino formation and decay requires that $\nu \chi_1^0$ decay into channels beyond the $\nu\chi_1^0$ formation channel. Informative examples include the parameter sets ($\Gamma$ and $L'$) presented among the post-WMAP benchmarks for the constrained minimal supersymmetric standard model in Ref. $\mathrm{11}$. (The same two models have been considered by Datta and collaborators $\mathrm{12}$ in their recent study of the Lorentz-boosted situation of ultrahigh-energy neutralinos scattering on the relic neutrino background.)

We summarize the relevant information in Table 1. The
TABLE I: Superpartner parameters in two MSSM scenarios \[11\], evaluated using the SPheno \[12\] code for particle properties and the MicrOMEGAs \[13\] code for relic densities, as implemented at the Comparison of SUSY spectrum generators web site \[14\] with \(m_1 = 172.5\) GeV \[10\] and the default values \(m_0 = 4.214\) GeV and \(\alpha_s(M_Z^2) = 0.1172\).

<table>
<thead>
<tr>
<th></th>
<th>Model I'</th>
<th>Model L'</th>
</tr>
</thead>
<tbody>
<tr>
<td>(M(\chi_1^0))</td>
<td>141 GeV</td>
<td>185 GeV</td>
</tr>
<tr>
<td>(\Omega_{\chi}h^2)</td>
<td>0.105</td>
<td>0.111</td>
</tr>
<tr>
<td>(N_{\chi}^{\ell})</td>
<td>(7.7 \times 10^{-9}) cm(^{-3})</td>
<td>(6.4 \times 10^{-9}) cm(^{-3})</td>
</tr>
<tr>
<td>(N_{\chi}^{\nu})</td>
<td>(2.2 \times 10^{-3}) cm(^{-3})</td>
<td>(1.6 \times 10^{-3}) cm(^{-3})</td>
</tr>
<tr>
<td>(M(\tilde{\chi}_1^0))</td>
<td>&lt; 400 cm(^{-3})</td>
<td>&lt; 300 cm(^{-3})</td>
</tr>
<tr>
<td>(m_{\tilde{\nu}_e,\mu})</td>
<td>265 GeV</td>
<td>265 GeV</td>
</tr>
<tr>
<td>(E_{\nu\chi})</td>
<td>299 GeV</td>
<td>422 GeV</td>
</tr>
<tr>
<td>(\Gamma_\chi)</td>
<td>324 MeV</td>
<td>481 MeV</td>
</tr>
<tr>
<td>(\sigma_{\nu\chi,\text{inel}}^{\text{peak}})</td>
<td>40.2 nb</td>
<td>20.6 nb</td>
</tr>
<tr>
<td>(\Gamma_\chi\sigma_{\nu\chi,\text{inel}}^{\text{peak}})</td>
<td>13.0 nb GeV</td>
<td>17.7 nb GeV</td>
</tr>
<tr>
<td>(B(\tilde{\nu} \rightarrow \chi_1^0\nu))</td>
<td>0.720</td>
<td>0.434</td>
</tr>
<tr>
<td>(B(\tilde{\nu} \rightarrow \chi_2^0\nu))</td>
<td>0.086</td>
<td>0.179</td>
</tr>
<tr>
<td>(B(\tilde{\nu} \rightarrow \chi_1^0\chi_1^-))</td>
<td>0.194</td>
<td>0.386</td>
</tr>
<tr>
<td>(m_{\tilde{\nu}_e})</td>
<td>277 GeV</td>
<td>386 GeV</td>
</tr>
<tr>
<td>(E_{\nu\chi}^{\text{res}})</td>
<td>272 GeV</td>
<td>403 GeV</td>
</tr>
<tr>
<td>(\Gamma_\chi)</td>
<td>612 MeV</td>
<td>2011 GeV</td>
</tr>
<tr>
<td>(\sigma_{\nu\chi,\text{inel}}^{\text{peak}})</td>
<td>52.3 nb</td>
<td>14.3 nb</td>
</tr>
<tr>
<td>(\Gamma_\chi\sigma_{\nu\chi,\text{inel}}^{\text{peak}})</td>
<td>32.0 nb GeV</td>
<td>28.8 nb GeV</td>
</tr>
<tr>
<td>(B(\tilde{\nu} \rightarrow \chi_1^0\nu_e))</td>
<td>0.342</td>
<td>0.153</td>
</tr>
<tr>
<td>(B(\tilde{\nu} \rightarrow \chi_2^0\nu_e))</td>
<td>0.012</td>
<td>0.023</td>
</tr>
<tr>
<td>(B(\tilde{\nu} \rightarrow \chi_1^0\tau^-))</td>
<td>0.026</td>
<td>0.052</td>
</tr>
<tr>
<td>(B(\tilde{\nu} \rightarrow \overline{\nu}\tau^+\overline{\nu}))</td>
<td>0.620</td>
<td>0.772</td>
</tr>
</tbody>
</table>

first point to notice is that the neutrino energy at resonance is approximately 300 and 500 GeV for the two superparticle spectra under study—squarely in the atmospheric neutrino range. Accordingly, there is no need to invoke unknown mechanisms to produce the required neutrino “beam,” as we must for the \(\nu\nu \rightarrow Z^0\) resonance. The modest resonant energies follow from the fact that the neutralino mass is more than eleven orders of magnitude larger than the (relic) neutrino mass. In both cases, inelastic processes—sneutrino decays that do not return to the entrance channel—are prominent.

There ends the good news, at least for the Universe at large. Whereas we expect that stable neutrinos should be the most abundant particles in the Universe after the photons of the cosmic microwave background, the neutralino gas is on average very tenuous. The neutralino fraction of the Universe (identified with the dark-matter fraction) is

\[
\Omega_{\chi}h^2 = \frac{\theta_{\xi}h^2}{\varrho_c},
\]

where the numerical value of the critical density is

\[
\varrho_c = 1.05 \times 10^{-5} h^2 \text{ GeV cm}^{-3}.
\]

Consequently the mass density of relic neutralinos is

\[
\theta_{\chi} = 1.05 \times 10^{-5} \cdot \Omega_{\chi}h^2 \text{ GeV cm}^{-3} = N_{\chi}M(\chi_1^0),
\]

where \(N_{\chi}\) is the mean number density of relic neutralinos throughout the Universe. In the two models we consider here, \(N_{\chi} \lesssim 10^{-8} \text{ cm}^{-3}\), some ten orders of magnitude smaller than the relic neutrino density and 31 orders of magnitude smaller than the density of electrons in water.

The peak cross sections for inelastic sneutrino formation

\[
\sigma_{\nu\chi,\text{inel}}^{\text{peak}} = \frac{8\pi m_{\tilde{\nu}}^2}{(m_{\tilde{\nu}}^2 - M_{\chi}^2)^2} B(\tilde{\nu} \rightarrow \chi_1^0\nu)[1 - B(\tilde{\nu} \rightarrow \chi_1^0\nu)],
\]

are about an order of magnitude smaller than that \[3\] for \(Z^0\) formation, \(\sigma(\nu\nu \rightarrow Z)^{\text{peak}}_{\text{visible}} = 0.80 \sigma(\nu\nu \rightarrow Z)^{\text{peak}}_{\text{inel}} = 365\) nb. The cross sections integrated over the peak are some fifty to seventy times smaller than \(\Gamma_Z\sigma(\nu\nu \rightarrow Z)^{\text{peak}}_{\text{visible}} = 912\) nb GeV, where \(\Gamma_Z = 2.4952\) GeV. At the resonance peak, the interaction length for neutrinos on neutralinos in the present Universe is

\[
L_{\text{int}}^{\nu} = \frac{1}{\sigma_{\nu\chi,\text{inel}}^{\text{peak}}} N_{\chi}.
\]

For the two spectra under consideration, we find \(L_{\text{int}}^{\nu} \approx (1.2, 2.4) \times 10^{16}\) Mpc for model \((\Gamma, L')\), for \(\nu e,\mu\) interactions, with similar values for \(\nu\tau\) interactions. These values are roughly 11 orders of magnitude longer than the interaction length at resonance for \(\nu\nu \rightarrow Z^0\) which is \(L_{\text{int}}^{\nu} \approx 1.2 \times 10^{14}\) Mpc in the current Universe \[8\]. A distance of \(10^{15}\) Mpc corresponds to approximately \(10^{21.5}\) light years, so exceeds the 1.3 \times 10^{10}-year age of the Universe by a prodigious factor. If \(\nu\chi_1^0 \rightarrow \tilde{\nu}\) coannihilation were to be observable, absorption lines would not be the signature! We should have to rely on the direct detection of few-GeV \(\gamma\) rays or charged particles from the inelastic decay chains that lead back to the \(\nu\chi_1^0\) ground state.

Our location in the Universe may not be privileged, but it is not average. Cold dark matter clusters in galaxies. Most of the analytic expressions proposed in the literature for dark-matter halo profiles can be cast in the form

\[
\theta_{\chi}(r) = \frac{\theta_0}{(r/a)^\gamma [1 + (r/a)^\alpha]^{(\beta-\gamma)/\alpha}},
\]

where \(\theta_0\) is a characteristic density, \(a\) sets the radius of the halo, \(\gamma\) is the asymptotic logarithmic slope at the center, \(\beta\) is the slope as \(r \rightarrow \infty\), and \(\alpha\) controls the detailed shape of the profile in the intermediate regions around
produce the presumed dark-matter density (neglecting enclosed in a sphere of radius $r_\odot$).

<table>
<thead>
<tr>
<th>Model</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$a$ [kpc]</th>
<th>$\varrho_0$ [GeV cm$^{-3}$]</th>
<th>$\mu_\odot$ [$10^{67}$ GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ISO</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>4.0</td>
<td>1.655</td>
<td>3.8847</td>
</tr>
<tr>
<td>BE [18]</td>
<td>1</td>
<td>3</td>
<td>0.3</td>
<td>10.2</td>
<td>1.459</td>
<td>3.8351</td>
</tr>
<tr>
<td>NFW [19]</td>
<td>1</td>
<td>3</td>
<td>1</td>
<td>16.7</td>
<td>0.347</td>
<td>4.4205</td>
</tr>
<tr>
<td>M99 [20]</td>
<td>1.5</td>
<td>3</td>
<td>1.5</td>
<td>29.5</td>
<td>0.0536</td>
<td>4.8675</td>
</tr>
</tbody>
</table>

TABLE II: Parameters of four radial density profiles of the Milky Way dark halo considered in Ref. [17], according to the parametrization given by expression (6), plus the mass enclosed in a sphere of radius $r_\odot$.

According to the calculations of the atmospheric neutrinos, which would imply upper bounds on the number density of neutralinos at Earth, $N_{\tilde{\chi}}(r_\odot) \approx \varrho_\odot(r_\odot)/M(\tilde{\chi})$, are given in Table II the density profiles themselves are shown in Figure 2 along with the dark mass enclosed within a sphere of specified galactocentric radius.

What little we know about the distribution of dark matter within the solar system has been deduced by investigation possible dark-matter perturbations on the orbits of planets and asteroids. In a recent contribution, Khriplovich and Pitjeva [22] have deduced bounds on the density for the Universe at large. The resulting neutralino number densities, $N_{\tilde{\chi}}(r_\odot) \approx \varrho_\odot(r_\odot)/M(\tilde{\chi})$, are given in Table II the density profiles themselves are shown in Figure 2 along with the dark mass enclosed within a sphere of specified galactocentric radius.

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The quantity $\Gamma_\nu \sigma_{\nu\text{-inel}}^{\text{peak}}$ provides a rough measure of the coannihilation cross section integrated over the resonance peak. Consequently, the product

$$\mathcal{R}_\nu = \Gamma_\nu \sigma_{\nu\text{-inel}}^{\text{peak}} \frac{dN_\nu}{dE_\nu}$$

characterizes the $\tilde{\nu}$ coannihilation rate of a single neutralino in the rain of atmospheric neutrinos.

For model I’, we compute the coannihilation rate $\mathcal{R}_\nu^{(I')} \approx 4.3 \times 10^{-40}$ s$^{-1}$. If we consider the neutralino density for the Universe at large, then the rate per unit target volume is

$$\mathcal{R}_\nu^{(I')} = N_{\tilde{\chi}} R_{\nu}^{(I')} \approx 3.3 \times 10^{-48} \text{ cm}^{-3} \text{ s}^{-1} \approx 10^{-40} \text{ cm}^{-3} \text{ y}^{-1},$$

which is laughably small, and smaller still for model I’.

To pursue this line to a logical conclusion, we ask how many interactions of atmospheric neutrinos with relic neutralinos occur in the atmosphere per unit time. Taking the Earth to be a sphere with radius $R_\odot = 6371$ km, we compute the volume of a 10-km-thick shell above the Earth’s surface to be $V_{\text{atm}} = 5.1 \times 10^{24}$ cm$^3$. The number of neutrino events induced in this volume is therefore $\lesssim 3 \times 10^{-5}$ y$^{-1}$, taking the largest conceivable neutralino number density. Such an infinitesimal rate renders moot a discussion of signatures and detection efficiencies. Setting aside questions of detection, the total number of
events within the Earth’s volume, \( V_\oplus = 1.08 \times 10^{27} \) cm\(^3\), is no more than \( 6 \times 10^{-3} \) y\(^{-1}\).

Extraterrestrial sources may interact with relic neutrinos over a larger volume. A representative estimate of the diffuse neutrino flux from active galactic nuclei suggests that, at \( E_\nu \approx 300 \) GeV, the flux of cosmic neutrinos arriving from all directions is

\[
\frac{dN_\nu}{dE_\nu} = 6.3 \times 10^{-17} \text{ cm}^{-2} \text{ s}^{-1} \text{ GeV}^{-1},
\]

approximately \( 2 \times 10^{-9} \times \) the vertical atmospheric neutrino flux we have just considered. The upper bound on the amount of dark matter in the sphere defined by Earth’s orbit, \( \mu(1 \text{ au}) < 7.85 \times 10^{14} \) GeV \[22\], could therefore be the site of up to \( (144,130) \nu X^0_1 \to \nu \) inelastic interactions per year for model \((\Gamma', L')\).

Let us compute the contribution of each event, at its own location, to the signal recorded by a detector of unit area at Earth. It will be sufficient to assume that the detector records signals from all relevant directions with perfect efficiency. We define the vector from the Sun to the detector as \( \vec{s} \equiv (0, 0, s) \), and denote the location of the target as \( \vec{r} \equiv r(\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \phi) \). Then the vector that points from the target to the detector is \( \vec{d} = \vec{s} - \vec{r} \), so that \( d^2 = r^2 + s^2 - 2rs \cos \theta \). The decay products of the sneutrino produced at \( \vec{r} \) are distributed isotropically. Accordingly, the fraction of the solid angle that the detector subtends.

The effective number of targets seen by a detector of unit area at \( \vec{s} \) is given by

\[
n_{\text{eff}}(s) = \frac{1}{4\pi M_\tilde{\chi}} \int d^3 \vec{r} \frac{\rho_\tilde{\chi}(r)}{d^2} = \frac{1}{2M_\tilde{\chi}} \int_0^s d(\cos \theta) \int_0^1 r^2 dr \rho_\tilde{\chi}(r)
\]

For the special case of a constant density \( \rho_\tilde{\chi} = 5.6 \times 10^4 \) GeV cm\(^{-3}\), and \( s = 1 \text{ au} = 1.496 \times 10^{13} \) cm, the effective number of targets is

\[
n_{\text{eff}}(s) = \frac{\rho_\tilde{\chi} s}{2M_\tilde{\chi}} \approx 3 \times 10^{15} \text{ cm}^{-2}.
\]

Using the cosmic-neutrino flux \[11\], we estimate that a detector in the vicinity of Earth would be sensitive to \( 7.7 \times 10^{-26} \) events cm\(^{-2}\) y\(^{-1}\). With such a small number of events, there is no hope of detecting the few-GeV gamma rays that would be created in the cascade back to \( X^0_1 \).

The halo of our galaxy contains a great quantity of dark matter: for the NFW profile, \( 4.4 \times 10^{67} \) GeV lies within the galactocentric radius of our solar system. The number of neutralino targets with which neutrinos might coannihilate to form sneutrinos is thus \( (3.1, 2.4) \times 10^{65} \) for model \((\Gamma', L')\). Focusing again on model \(\Gamma\) and taking the estimate \[11\] for the cosmic-neutrino flux, we find that the neutrino formation rate throughout the galaxy is \( 2.6 \times 10^{17} \text{ s}^{-1} = 8.1 \times 10^{24} \text{ y}^{-1}\). These rates are prodigious in absolute terms, but represent an insignificant disturbance to the galaxy’s neutralino population.

Can we hope to observe the sneutrino formation that might be bubbling away in our neighborhood? in this case, we define \( \vec{s} = (0, 0, s) \) to be the vector from the galactic center to the detector, and use \[12\] to compute the effective number of targets seen from Earth. For the NFW profile, we find \( n_{\text{eff}}(r_\odot) = 4.9 \times 10^{19} \) cm\(^{-2}\). For a detector of any plausible area, only a tiny fraction of the \( 3.1 \times 10^{65} \) neutralinos in the halo produce a visible signal. Indeed, using the neutrino flux \[11\], we expect that a detector in the vicinity of Earth would be sensitive to \( 1.3 \times 10^{-21} \) events cm\(^{-2}\) y\(^{-1}\).

It is not obvious that Earth’s location should happen to be optimal for viewing coannihilation events in the galaxy. In Figure 3 we show how the effective number of targets viewed depends on \( s \), the galactocentric radius of the observation point. For the smooth ISO and BE profiles, our solar system lies near the optimal distance, while for the NFW profile the effective number of targets viewed is insensitive to the detector position. The most singular profile we consider, M99, favors an observatory close to the galactic center—not that we could contemplate one—but even in this case, the sensitivity is enhanced by less than two orders of magnitude, which is far too little to enable detection. Singular profiles have more effect on the rates for neutralino-neutralino annihilations, which are proportional to the square of the
neutralino density.

If neutralinos account for much of the cold dark matter of the Universe, then it is possible that neutrino–neutralino coannihilation into sneutrinos is happening all around us, but at a rate that will forever elude detection.

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