

# Direct test of time-reversal symmetry in the entangled neutral kaon system at a $\phi$ -factory

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## Abstract

We present a novel method to perform a direct  $\mathcal{T}$  (time reversal) symmetry test in the neutral kaon system, independent of any  $\mathcal{CP}$  and/or  $\mathcal{CPT}$  symmetry tests. This is based on the comparison of suitable transition probabilities, where the required interchange of *in*  $\leftrightarrow$  *out* states for a given process is obtained exploiting the Einstein-Podolsky-Rosen correlations of neutral kaon pairs produced at a  $\phi$ -factory. In the time distribution between the two decays, we compare a reference transition like the one defined by the time ordered decays ( $\ell^-$ ,  $\pi\pi$ ) with the  $\mathcal{T}$ -conjugated one defined by ( $3\pi^0$ ,  $\ell^+$ ). With the use of this and other  $\mathcal{T}$  conjugated comparisons, the KLOE-2 experiment at DAΦNE could make a significant test.

*Keywords:*

Time reversal violation, Discrete Symmetries, Neutral Kaons,  $\phi$ -factory

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## 1. Introduction

$\mathcal{CP}$  violation in the Standard Model (SM) arises from the single physically relevant phase in the three families Cabibbo-Kobayashi-Maskawa (CKM)

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mixing matrix. The existence of this matrix conveys the fact that the quarks that participate in weak processes are a linear combination of mass eigenstates. This mechanism has been validated in the past years of experiments probing  $\mathcal{CP}$  violation, especially in K [1, 2] and B [3, 4] meson decays. In the context of local quantum field theories with Lorentz invariance and Hermiticity, the  $\mathcal{CPT}$  theorem ensures an automatic theoretical connection between  $\mathcal{CP}$  symmetry and  $\mathcal{T}$  (time reversal) symmetry. Since the SM is  $\mathcal{CPT}$  invariant, it predicts  $\mathcal{T}$  violating effects in parallel to each  $\mathcal{CP}$ -violation effect that arises due to the interference of amplitudes with different weak phases.

Even though  $\mathcal{CPT}$  invariance has been confirmed by all present experimental tests, particularly in the neutral kaon system where there are strong limits to possible  $\mathcal{CPT}$  violation effects [5, 6, 7, 8, 9, 10], the theoretical connection between  $\mathcal{CP}$  and  $\mathcal{T}$  symmetries does not imply an experimental identity between them, except for processes which are  $\mathcal{CPT}$  even, e.g.  $K^0 \rightarrow \bar{K}^0$  [11]. Therefore it is of great interest to search for direct evidence of non-invariance under time reversal, independent of  $\mathcal{CP}$  violation and  $\mathcal{CPT}$  invariance. Only recently, the first direct observation of  $\mathcal{T}$  violation, in this sense, has been accomplished in the neutral B meson system [12]. In the case of transition processes a test of  $\mathcal{T}$  non-invariance needs the comparison between the transition amplitudes under the interchange between *in* states and *out* states. For unstable systems, the associated irreversibility looks like it prevents a true test of  $\mathcal{T}$  symmetry [13].

In this article we describe the methodology to perform a direct test of  $\mathcal{T}$  symmetry in the neutral K meson system at a  $\phi$ -factory, overcoming the irreversibility problem, similarly as described in Ref. [14] for a B-factory. This methodology makes use of Einstein-Podolski-Rosen (EPR) entanglement [15], and relies on the possibility of preparing the quantum mechanical individual state of the neutral K meson by the observation of particular decay channels of its orthogonal entangled partner, and studying the time evolution of the filtered state of the still living meson. This strategy allows the interchange of *in*  $\leftrightarrow$  *out* states for a given process, as needed for a genuine test of  $\mathcal{T}$  symmetry. Whereas the basic ideas have been presented previously [16] and scrutinized later [13, 17, 18, 19], the discussion of the steps to implement these concepts into a B-factory experiment able to produce the desired result has been recently presented [14] and later actually observed in the neutral B meson system [12]. Here we discuss the corresponding concepts needed for a direct  $\mathcal{T}$  symmetry test in the physical context of the neutral *K* meson system at a  $\phi$ -factory. In addition we evaluate the statistical significance of

the test achievable with the KLOE-2 experiment at DAΦNE, the Frascati  $\phi$ -factory [20].

## 2. The kaon states

In order to formulate a possible  $\mathcal{T}$  symmetry test with neutral kaons, it is necessary to precisely define the different states involved. First, let us consider the physical states  $|K_S\rangle$ ,  $|K_L\rangle$ , i.e. the states with definite masses  $m_{S,L}$  and lifetimes  $\tau_{S,L}$  which evolve as a function of the kaon proper time  $t$  as pure exponentials

$$\begin{aligned} |K_S(t)\rangle &= e^{-i\lambda_S t} |K_S\rangle \\ |K_L(t)\rangle &= e^{-i\lambda_L t} |K_L\rangle . \end{aligned} \quad (1)$$

with  $\lambda_{S,L} = m_{S,L} - i\Gamma_{S,L}/2$ , and  $\Gamma_{S,L} = (\tau_{S,L})^{-1}$ . They are usually expressed in terms of the flavor eigenstates  $|K^0\rangle$ ,  $|\bar{K}^0\rangle$  as:

$$|K_S\rangle = \frac{1}{\sqrt{2(1+|\epsilon_S|^2)}} [(1+\epsilon_S)|K^0\rangle + (1-\epsilon_S)|\bar{K}^0\rangle] \quad (2)$$

$$|K_L\rangle = \frac{1}{\sqrt{2(1+|\epsilon_L|^2)}} [(1+\epsilon_L)|K^0\rangle - (1-\epsilon_L)|\bar{K}^0\rangle] , \quad (3)$$

with  $\epsilon_S$  and  $\epsilon_L$  two small complex parameters describing the  $\mathcal{CP}$  impurity in the physical states. One can equivalently define  $\epsilon \equiv (\epsilon_S + \epsilon_L)/2$ , and  $\delta \equiv (\epsilon_S - \epsilon_L)/2$ ; adopting a suitable phase convention (e.g. the Wu-Yang phase convention [23])  $\epsilon \neq 0$  implies  $\mathcal{T}$  violation,  $\delta \neq 0$  implies  $\mathcal{CP}\mathcal{T}$  violation, while  $\delta \neq 0$  or  $\epsilon \neq 0$  implies  $\mathcal{CP}$  violation.

Let us also consider the states  $|K_+\rangle$ ,  $|K_-\rangle$  defined as follows:  $|K_+\rangle$  is the state filtered by the decay into  $\pi\pi$  ( $\pi^+\pi^+$  or  $\pi^0\pi^0$ ), a pure  $\mathcal{CP} = +1$  state;  $|\tilde{K}_-\rangle$  is the state orthogonal to  $|K_+\rangle$ , i.e.  $\langle\tilde{K}_-|K_+\rangle = 0$ , which cannot decay into  $\pi\pi$ ,  $\langle\pi\pi|T|\tilde{K}_-\rangle = 0$ , and is defined by [24]:

$$|\tilde{K}_-\rangle \equiv \tilde{N}_- [|K_L\rangle - \eta_{\pi\pi}|K_S\rangle] \quad (4)$$

where  $\eta_{\pi\pi} = \frac{\langle\pi\pi|T|K_L\rangle}{\langle\pi\pi|T|K_S\rangle}$ , and  $|\tilde{N}_-|^2 = [1 + |\eta_{\pi\pi}|^2 - 2\Re(\eta_{\pi\pi}\langle K_L|K_S\rangle)]^{-1}$  defines the normalization constant up to a phase factor. Therefore the state  $|K_+\rangle$  can be explicitly written as the state orthogonal to  $|\tilde{K}_-\rangle$  as:

$$|K_+\rangle = N_+ [|K_S\rangle + \alpha|K_L\rangle] \quad (5)$$

where

$$\alpha = \frac{\eta_{\pi\pi}^* - \langle K_L | K_S \rangle}{1 - \eta_{\pi\pi}^* \langle K_S | K_L \rangle}, \quad (6)$$

and  $|N_+|^2 = [1 + |\alpha|^2 + 2\Re(\alpha \langle K_S | K_L \rangle)]^{-1}$ .

Analogously  $|K_- \rangle$  is the state filtered by the decay into  $3\pi^0$ , a pure  $\mathcal{CP} = -1$  state;  $|\tilde{K}_+ \rangle$  is the state orthogonal to  $|K_- \rangle$ , i.e.  $\langle \tilde{K}_+ | K_- \rangle = 0$ , which cannot decay into  $3\pi^0$ ,  $\langle \pi\pi | T | \tilde{K}_- \rangle = 0$ , and is defined by:

$$|\tilde{K}_+ \rangle \equiv \tilde{N}_+ [ |K_S \rangle - (\eta_{3\pi^0}^{-1}) |K_L \rangle ] \quad (7)$$

where  $(\eta_{3\pi^0}^{-1}) = \frac{\langle 3\pi^0 | T | K_S \rangle}{\langle 3\pi^0 | T | K_L \rangle}$ , and  $|\tilde{N}_+|^2 = [1 + |(\eta_{3\pi^0}^{-1})|^2 - 2\Re((\eta_{3\pi^0}^{-1})^* \langle K_L | K_S \rangle)]^{-1}$ . Therefore the state  $|K_- \rangle$  can be explicitly written as the state orthogonal to  $|\tilde{K}_+ \rangle$  as:

$$|K_- \rangle = N_- [ |K_L \rangle + \beta |K_S \rangle ] \quad (8)$$

where

$$\beta = \frac{(\eta_{3\pi^0}^{-1})^* - \langle K_S | K_L \rangle}{1 - (\eta_{3\pi^0}^{-1})^* \langle K_L | K_S \rangle}, \quad (9)$$

and  $|N_-|^2 = [1 + |\beta|^2 + 2\Re(\beta \langle K_L | K_S \rangle)]^{-1}$ .

Even though in the following we will assume that

$$\begin{aligned} |K_+ \rangle &\equiv |\tilde{K}_+ \rangle \\ |K_- \rangle &\equiv |\tilde{K}_- \rangle, \end{aligned} \quad (10)$$

here we have kept separate definitions of the states  $|K_+ \rangle$  and  $|K_- \rangle$ , which are observed through their decay, from the states  $|\tilde{K}_+ \rangle$  and  $|\tilde{K}_- \rangle$ , which are produced exploiting the EPR correlations in entangled kaon pairs, as we will see in the next section.

Assumption (10) corresponds to impose the condition of orthogonality  $\langle K_- | K_+ \rangle = 0$  or  $\langle \tilde{K}_- | \tilde{K}_+ \rangle = 0$ . This implies that  $\beta = -\eta_{\pi\pi}$  and  $\alpha = -(\eta_{3\pi^0}^{-1})$ , which in turn imply a precise relationship between the two amplitude ratios  $\eta_{\pi\pi}$  and  $(\eta_{3\pi^0}^{-1})$ , i.e.:

$$\begin{aligned} \eta_{\pi\pi} &= \frac{\langle K_S | K_L \rangle - (\eta_{3\pi^0}^{-1})^*}{1 - (\eta_{3\pi^0}^{-1})^* \langle K_L | K_S \rangle} \\ &\simeq \langle K_S | K_L \rangle - (\eta_{3\pi^0}^{-1})^*, \end{aligned} \quad (11)$$

or put in another form:

$$\eta_{\pi\pi} + (\eta_{3\pi^0}^{-1})^* \simeq \langle K_S | K_L \rangle \simeq \epsilon_L + \epsilon_S^* . \quad (12)$$

This equation clearly indicates that we have to neglect direct  $\mathcal{CP}$  violation when imposing assumption (10). In fact, for instance, eq.(12) cannot be simultaneously satisfied for  $\pi^+\pi^+$  and  $\pi^0\pi^0$  decays, being  $(\eta_{\pi^+\pi^+} - \eta_{\pi^0\pi^0}) = 3\epsilon'$ , with  $\epsilon'$  the direct  $\mathcal{CP}$  violation parameter [6].

The relevance of this assumption will be discussed in Appendix A, where it will be shown that direct  $\mathcal{CP}$  violation can be safely neglected for our purposes.

Finally we will assume the validity of the  $\Delta S = \Delta Q$  rule, so that the two flavor orthogonal eigenstates  $|K^0\rangle$  and  $|\bar{K}^0\rangle$  are identified by the charge of the lepton in semileptonic decays, i.e. a  $|K^0\rangle$  can decay into  $\pi^-\ell^+\nu$  and not into  $\pi^+\ell^-\bar{\nu}$ , and vice-versa for a  $|\bar{K}^0\rangle$ .

### 3. Observables for the $\mathcal{T}$ symmetry test

A direct evidence of  $\mathcal{T}$  violation would mean an experiment that, considered by itself, clearly shows the violation independent of and unconnected to the results of  $\mathcal{CP}$  violation. There is no existing result in the neutral K system that clearly demonstrates time reversal violation in this sense [13]. Sometimes the Kabir asymmetry  $K^0 \rightarrow \bar{K}^0$  vs.  $\bar{K}^0 \rightarrow K^0$  has been presented [21, 22, 18] as a proof for  $\mathcal{T}$  violation. This process has, however, besides the drawbacks discussed in [13], the feature that  $K^0 \rightarrow \bar{K}^0$  is a  $\mathcal{CPT}$  even transition, so that it is impossible to separate  $\mathcal{T}$  violation from  $\mathcal{CP}$  violation in the Kabir asymmetry: these two transformations are experimentally identical in this case.

There are effects in particle physics that are odd under time  $t \rightarrow -t$ , but they are not genuine violations of time reversal  $\mathcal{T}$ , because do not correspond to an interchange of *in*-states into *out*-states. These kinds of  $t$ -asymmetries, like the macroscopic and the Universe  $t$ -asymmetry, can occur in theories which have an exact  $\mathcal{T}$  symmetry in the underlying fundamental physics [17]. In fact, the  $t$ -asymmetry can only be connected [16] to  $\mathcal{T}$  asymmetry under the assumptions of  $\mathcal{CPT}$  invariance plus the absence of an absorptive part difference between the initial and final states of the transition. As a consequence, we have to disregard these  $t$ -asymmetries as direct evidence for  $\mathcal{T}$  violation.

As shown in [16, 19], B-factories and  $\phi$ -factories offer the unique opportunity to show evidence for  $\mathcal{T}$  violation (and  $\mathcal{CP}$  violation) independently from the other symmetries and to measure the corresponding effects. The EPR entanglement here plays a crucial role. Let us consider the neutral kaon pair produced at a  $\phi$ -factory in a coherent quantum state with quantum numbers  $J^{PC} = 1^{--}$  [26]:

$$|i\rangle = \frac{1}{\sqrt{2}}\{ |K^0\rangle|\bar{K}^0\rangle - |\bar{K}^0\rangle|K^0\rangle \} \quad (13)$$

$$= \frac{1}{\sqrt{2}}\{ |K_+\rangle|K_-\rangle - |K_-\rangle|K_+\rangle \} . \quad (14)$$

It's worth noting that one can rewrite the two particle state  $|i\rangle$  in terms of any pair of orthogonal states of individual neutral K mesons, e.g.,  $K^0$  and  $\bar{K}^0$ , or  $K_+$  and  $K_-$  defined in section 2. The time evolution of the initial state is simply given by  $|i(t)\rangle = e^{-i(\lambda_S + \lambda_L)t}|i\rangle$ , with  $t$  common proper time of the two kaons; the initial EPR correlation given by  $|i\rangle$  remains unaltered until one of the two kaons decays. One has also to emphasize, following what quantum mechanics dictates, that the individual state of one neutral meson in the entangled state is not defined before the decay process of its partner occurs, imposing a tag over the undecayed kaon. Thus it is possible to have a “flavor-tag”, i.e. to infer the flavor ( $K^0$  or  $\bar{K}^0$ ) of the still alive meson by observing the specific flavor decay ( $\pi^+\ell^-\bar{\nu}$  or  $\pi^-\ell^+\nu$ ) of the other (and first decaying) meson. Similarly we may define a “ $\mathcal{CP}$ -tag” [25] as the filter imposed by the decay of one of the entangled states to a  $K_+$  or  $K_-$ , preparing its partner, which has not decayed yet, into the orthogonal state  $K_-$  or  $K_+$ , respectively. In this way we may proceed to a partition of the complete set of events into four categories, defined by the tag in the first decay as  $K_+$ ,  $K_-$ ,  $K^0$  or  $\bar{K}^0$ .

Let us first consider  $K^0 \rightarrow K_+$  as the reference process, by observation of a  $\pi^+\ell^-\bar{\nu}$  decay at a proper time  $t_1$  of the opposite  $\bar{K}^0$  meson<sup>1</sup> and a  $\pi\pi$  decay at a later time  $t_2 > t_1$ , denoted as  $(\ell^-, \pi\pi)$ , and consider:

- i) Its  $\mathcal{T}$  transformed  $K_+ \rightarrow K^0$  ( $3\pi^0, \ell^+$ ), so that the asymmetry between  $K^0 \rightarrow K_+$  and  $K_+ \rightarrow K^0$ , as a function of  $\Delta t = t_2 - t_1$ , is a genuine  $\mathcal{T}$  violating effect.

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<sup>1</sup>To relax the notation we will denote  $\pi^+\ell^-\bar{\nu}$  as  $\ell^-$  and  $\pi^-\ell^+\nu$  as  $\ell^+$ , because of the lepton charge.

- ii) Its  $\mathcal{CP}$  transformed  $\bar{K}^0 \rightarrow K_+$  ( $\ell^+, \pi\pi$ ), so that the asymmetry between  $K^0 \rightarrow K_+$  and  $\bar{K}^0 \rightarrow K_+$ , as a function of  $\Delta t = t_2 - t_1$ , is a genuine  $\mathcal{CP}$  violating effect.
- iii) Its  $\mathcal{CPT}$  transformed  $K_+ \rightarrow \bar{K}^0$  ( $3\pi^0, \ell^-$ ), so that the asymmetry between  $K^0 \rightarrow K_+$  and  $K_+ \rightarrow \bar{K}^0$ , as a function of  $\Delta t = t_2 - t_1$ , is a genuine test of  $\mathcal{CPT}$  invariance.

One may check, that the events used for the asymmetries i), ii), and iii) are completely independent.

There are other three independent comparisons between  $\mathcal{T}$ -conjugated processes, as summarized in Table 1. Analogously, we can apply the same methodology for similar tests of  $\mathcal{CP}$  violation and  $\mathcal{CPT}$  invariance. Tables 2 and 3 summarize all the possible comparisons of  $\mathcal{CP}$ - and  $\mathcal{CPT}$ -conjugated transitions with their corresponding decay products.

Reference		$\mathcal{T}$ -conjugate	
Transition	Decay products	Transition	Decay products
$K^0 \rightarrow K_+$	$(\ell^-, \pi\pi)$	$K_+ \rightarrow K^0$	$(3\pi^0, \ell^+)$
$K^0 \rightarrow K_-$	$(\ell^-, 3\pi^0)$	$K_- \rightarrow K^0$	$(\pi\pi, \ell^+)$
$\bar{K}^0 \rightarrow K_+$	$(\ell^+, \pi\pi)$	$K_+ \rightarrow \bar{K}^0$	$(3\pi^0, \ell^-)$
$\bar{K}^0 \rightarrow K_-$	$(\ell^+, 3\pi^0)$	$K_- \rightarrow \bar{K}^0$	$(\pi\pi, \ell^-)$

Table 1: Possible comparisons between  $\mathcal{T}$ -conjugated transitions and the associated time-ordered decay products in the experimental  $\phi$ -factory scheme.

Reference		$\mathcal{CP}$ -conjugate	
Transition	Decay products	Transition	Decay products
$K^0 \rightarrow K_+$	$(\ell^-, \pi\pi)$	$\bar{K}^0 \rightarrow K_+$	$(\ell^+, \pi\pi)$
$K^0 \rightarrow K_-$	$(\ell^-, 3\pi^0)$	$\bar{K}^0 \rightarrow K_-$	$(\ell^+, 3\pi^0)$
$\bar{K}^0 \rightarrow K_+$	$(\ell^+, \pi\pi)$	$K^0 \rightarrow K_+$	$(\ell^-, \pi\pi)$
$\bar{K}^0 \rightarrow K_-$	$(\ell^+, 3\pi^0)$	$K^0 \rightarrow K_-$	$(\ell^-, 3\pi^0)$

Table 2: Possible comparisons between  $\mathcal{CP}$ -conjugated transitions and the associated time-ordered decay products in the experimental  $\phi$ -factory scheme.

Reference		$\mathcal{CPT}$ -conjugate	
Transition	Decay products	Transition	Decay products
$K^0 \rightarrow K_+$	$(\ell^-, \pi\pi)$	$K_+ \rightarrow \bar{K}^0$	$(3\pi^0, \ell^-)$
$K^0 \rightarrow K_-$	$(\ell^-, 3\pi^0)$	$K_- \rightarrow \bar{K}^0$	$(\pi\pi, \ell^-)$
$\bar{K}^0 \rightarrow K_+$	$(\ell^+, \pi\pi)$	$K_+ \rightarrow K^0$	$(3\pi^0, \ell^+)$
$\bar{K}^0 \rightarrow K_-$	$(\ell^+, 3\pi^0)$	$K_- \rightarrow K^0$	$(\pi\pi, \ell^+)$

Table 3: Possible comparisons between  $\mathcal{CPT}$ -conjugated transitions and the associated time-ordered decay products in the experimental  $\phi$ -factory scheme.

Our goal is to demonstrate and measure the violation of time reversal invariance. Therefore we have to consider the following ratios of probabilities:

$$\begin{aligned}
R_1(\Delta t) &= P [K^0(0) \rightarrow K_+(\Delta t)] / P [K_+(0) \rightarrow K^0(\Delta t)] \\
R_2(\Delta t) &= P [K^0(0) \rightarrow K_-(\Delta t)] / P [K_-(0) \rightarrow K^0(\Delta t)] \\
R_3(\Delta t) &= P [\bar{K}^0(0) \rightarrow K_+(\Delta t)] / P [K_+(0) \rightarrow \bar{K}^0(\Delta t)] \\
R_4(\Delta t) &= P [\bar{K}^0(0) \rightarrow K_-(\Delta t)] / P [K_-(0) \rightarrow \bar{K}^0(\Delta t)] . \quad (15)
\end{aligned}$$

The measurement of any deviation from the prediction

$$R_1(\Delta t) = R_2(\Delta t) = R_3(\Delta t) = R_4(\Delta t) = 1 \quad (16)$$

imposed by  $\mathcal{T}$  invariance is a signal of  $\mathcal{T}$  violation. This outcome will be highly rewarding as a model-independent and a direct observation of  $\mathcal{T}$  violation.

If we express two generic orthogonal basis  $\{K_X, \bar{K}_X\}$  and  $\{K_Y, \bar{K}_Y\}$ , which in our case correspond to  $\{K^0, \bar{K}^0\}$  or  $\{K_+, K_-\}$ , as follows:

$$|K_X\rangle = X_S|K_S\rangle + X_L|K_L\rangle \quad (17)$$

$$|\bar{K}_X\rangle = \bar{X}_S|K_S\rangle + \bar{X}_L|K_L\rangle \quad (18)$$

$$|K_Y\rangle = Y_S|K_S\rangle + Y_L|K_L\rangle \quad (19)$$

$$|\bar{K}_Y\rangle = \bar{Y}_S|K_S\rangle + \bar{Y}_L|K_L\rangle . \quad (20)$$

the generic quantum mechanical expression for the probabilities entering

in eqs.(15) is given by

$$\begin{aligned}
P [K_X(0) \rightarrow K_Y(\Delta t)] &= |\langle K_Y | K_X(\Delta t) \rangle|^2 & (21) \\
&= \frac{1}{|\det Y|^2} |e^{-i\lambda_S \Delta t} X_S \bar{Y}_L - e^{-i\lambda_L \Delta t} X_L \bar{Y}_S|^2 \\
&= \frac{1}{|\det Y|^2} \{e^{-\Gamma_S \Delta t} |X_S \bar{Y}_L|^2 + e^{-\Gamma_L \Delta t} |X_L \bar{Y}_S|^2 \\
&\quad - 2e^{-\frac{(\Gamma_S + \Gamma_L)}{2} \Delta t} \Re (e^{i\Delta m \Delta t} X_S \bar{Y}_L X_L^* \bar{Y}_S^*)\} ,
\end{aligned}$$

with

$$\det Y = Y_S \bar{Y}_L - Y_L \bar{Y}_S \quad (22)$$

and

$$|\det Y|^2 = |\det X|^2 = \frac{1}{1 - |\langle K_S | K_L \rangle|^2} . \quad (23)$$

Its inverse  $P [K_Y(0) \rightarrow K_X(\Delta t)]$  is obtained simply with the substitution  $X \leftrightarrow Y$ .

Using the expected values for the  $X_{S,L}$ ,  $\bar{X}_{S,L}$ ,  $Y_{S,L}$  and  $\bar{Y}_{S,L}$  coefficients in terms of the measured  $\epsilon$  and  $\delta$  parameters [6], it can be easily demonstrated that the ratios  $R_i$  depend on  $\Delta t$ , as it is shown in Fig.1. This result is in contrast with the Kabir  $\mathcal{T}$ -violating asymmetry [11, 21, 22], which is independent of time:

$$\begin{aligned}
\frac{P [K^0(0) \rightarrow \bar{K}^0(\Delta t)]}{P [\bar{K}^0(0) \rightarrow K^0(\Delta t)]} &= \frac{|X_S X_L e^{-i\lambda_S \Delta t} - X_L X_S e^{-i\lambda_L \Delta t}|^2}{|\bar{X}_S \bar{X}_L e^{-i\lambda_S \Delta t} - \bar{X}_L \bar{X}_S e^{-i\lambda_L \Delta t}|^2} = \frac{|X_S X_L|^2}{|\bar{X}_S \bar{X}_L|^2} \\
&\simeq \frac{(1 - 4\Re\epsilon)}{(1 + 4\Re\epsilon)} \simeq 1 - 8\Re\epsilon . & (24)
\end{aligned}$$

It is worth noting that for  $\Delta t = 0$  we have:

$$R_1(0) = R_2(0) = R_3(0) = R_4(0) = 1 \quad (25)$$

and for  $\Delta t \gg \tau_S$  :

$$R_2(\Delta t \gg \tau_S) \simeq \frac{1 - 2\Re\epsilon_S}{1 + 2\Re\epsilon_L} \simeq 1 - 4\Re\epsilon \quad (26)$$

$$R_4(\Delta t \gg \tau_S) \simeq \frac{1 + 2\Re\epsilon_S}{1 - 2\Re\epsilon_L} \simeq 1 + 4\Re\epsilon \quad (27)$$

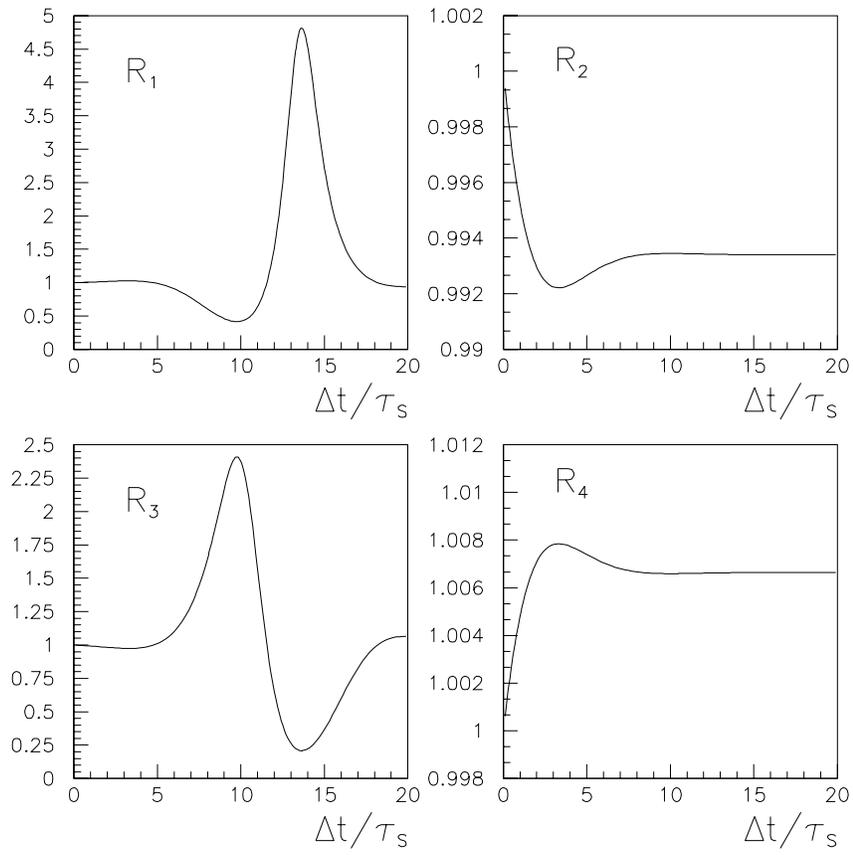


Figure 1: The ratios  $R_i$  as a function of  $\Delta t$ ;  $R_1$  top left,  $R_2$  top right,  $R_3$  bottom left,  $R_4$  bottom right .

#### 4. Measurement of $R_i$ at a $\phi$ -factory

From the experimental point of view the observable quantity at a  $\phi$ -factory is the double differential decay rate of the state  $|i\rangle$  into decay products  $f_1$  and  $f_2$  at proper times  $t_1$  and  $t_2$ , respectively [26]. For the time evolution of the system it is convenient to rewrite the entangled state  $|i\rangle$  as:

$$|i\rangle = \frac{\mathcal{N}}{\sqrt{2}}\{|\text{K}_S\rangle|\text{K}_L\rangle - |\text{K}_L\rangle|\text{K}_S\rangle\} \quad (28)$$

with  $|\mathcal{N}|^2 = |\det X|^2 = [(1 + |\epsilon_S|^2)(1 + |\epsilon_L|^2)]/(1 - \epsilon_S\epsilon_L)^2 \simeq 1$  a normalization factor. The double differential decay rate is given by:

$$I(f_1, t_1; f_2, t_2) = C_{12}\{|\eta_1|^2 e^{-\Gamma_L t_1 - \Gamma_S t_2} + |\eta_2|^2 e^{-\Gamma_S t_1 - \Gamma_L t_2} - 2|\eta_1||\eta_2| e^{-\frac{(\Gamma_S + \Gamma_L)}{2}(t_1 + t_2)} \cos[\Delta m(t_1 - t_2) + \phi_2 - \phi_1]\} \quad (29)$$

where

$$\eta_i \equiv |\eta_i| e^{i\phi_i} = \frac{\langle f_i | T | \text{K}_L \rangle}{\langle f_i | T | \text{K}_S \rangle}, \quad (30)$$

$$C_{12} = \frac{|\mathcal{N}|^2}{2} |\langle f_1 | T | \text{K}_S \rangle \langle f_2 | T | \text{K}_S \rangle|^2.$$

After integration on  $t_1$  at fixed time difference  $\Delta t = t_2 - t_1 > 0$ , the decay intensity (29) can be rewritten in a more suitable form, putting in evidence the probabilities we are aiming for. In particular it will be a function of the first decay product  $f_1 = f_{\bar{X}}$  (which takes place at time  $t_1$ , identifies a  $\bar{\text{K}}_X$  state, and tags a  $\text{K}_X$  state on the opposite side), the second decay products  $f_2 = f_Y$  (which takes place at time  $t_2$  and identifies a  $\text{K}_Y$  state):

$$\begin{aligned} I(f_{\bar{X}}, f_Y; \Delta t) &= \int_0^\infty I(f_{\bar{X}}, t_1; f_Y; t_2) dt_1 \\ &= \frac{1}{\Gamma_S + \Gamma_L} |\langle \text{K}_X \bar{\text{K}}_X | i \rangle \langle f_{\bar{X}} | T | \bar{\text{K}}_X \rangle \langle \text{K}_Y | \text{K}_X(\Delta t) \rangle \langle f_Y | T | \text{K}_Y \rangle|^2 \\ &= C(f_{\bar{X}}, f_Y) \times P[\text{K}_X(0) \rightarrow \text{K}_Y(\Delta t)], \end{aligned} \quad (31)$$

where the coefficient  $C(f_{\bar{X}}, f_Y)$ , depending only on the final states  $f_{\bar{X}}$  and

$f_Y$ , is given by:

$$\begin{aligned}
C(f_{\bar{X}}, f_Y) &= \frac{1}{2(\Gamma_S + \Gamma_L)} |\langle f_{\bar{X}}|T|\bar{K}_X\rangle \langle f_Y|T|K_Y\rangle|^2 \\
&= \frac{|\langle f_{\bar{X}}|T|K_S\rangle|^2 |\langle f_Y|T|K_S\rangle|^2}{2(\Gamma_S + \Gamma_L)} \times |(\bar{X}_S + \eta_{\bar{X}}\bar{X}_L)(Y_S + \eta_Y Y_L)|^2,
\end{aligned} \tag{32}$$

and the generic probability  $P [K_X(0) \rightarrow K_Y(\Delta t)]$ , containing the only time dependence, is the one given by eq.(21).

From eq.(32) and the condition:

$$\begin{aligned}
I(f_{\bar{X}}, f_Y; \Delta t = 0) &= C(f_{\bar{X}}, f_Y) \times |\langle K_Y|K_X\rangle|^2 \\
&= C(f_Y, f_{\bar{X}}) \times |\langle \bar{K}_X|\bar{K}_Y\rangle|^2,
\end{aligned} \tag{33}$$

it can be easily shown that the coefficient  $C(f_{\bar{X}}, f_Y)$  is invariant under interchange  $f_Y \leftrightarrow f_{\bar{X}}$ , i.e.

$$C(f_{\bar{X}}, f_Y) = C(f_Y, f_{\bar{X}}). \tag{34}$$

One can define the following observable ratios:

$$R_1^{\text{exp}}(\Delta t) \equiv \frac{I(\ell^-, \pi\pi; \Delta t)}{I(3\pi^0, \ell^+; \Delta t)} = R_1(\Delta t) \times \frac{C(\ell^-, \pi\pi)}{C(3\pi^0, \ell^+)} \tag{35}$$

$$R_2^{\text{exp}}(\Delta t) \equiv \frac{I(\ell^-, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^+; \Delta t)} = R_2(\Delta t) \times \frac{C(\ell^-, 3\pi^0)}{C(\pi\pi, \ell^+)} \tag{36}$$

$$R_3^{\text{exp}}(\Delta t) \equiv \frac{I(\ell^+, \pi\pi; \Delta t)}{I(3\pi^0, \ell^-; \Delta t)} = R_3(\Delta t) \times \frac{C(\ell^+, \pi\pi)}{C(3\pi^0, \ell^-)} \tag{37}$$

$$R_4^{\text{exp}}(\Delta t) \equiv \frac{I(\ell^+, 3\pi^0; \Delta t)}{I(\pi\pi, \ell^-; \Delta t)} = R_4(\Delta t) \times \frac{C(\ell^+, 3\pi^0)}{C(\pi\pi, \ell^-)}, \tag{38}$$

which are proportional to the corresponding  $R_i(\Delta t)$  ratios.

It should be noted that when we perform a measurement with decay products in inverse time order ( $f_2, f_1$ ) or  $t_1 > t_2$ , i.e.  $\Delta t \rightarrow -\Delta t$ , we are actually measuring the inverse of *another* ratio, i.e.:

$$R_2^{\text{exp}}(-\Delta t) = \frac{1}{R_3^{\text{exp}}(\Delta t)} = \frac{1}{R_3(\Delta t)} \times \frac{C(3\pi^0, \ell^-)}{C(\ell^+, \pi\pi)} \quad (39)$$

$$R_4^{\text{exp}}(-\Delta t) = \frac{1}{R_1^{\text{exp}}(\Delta t)} = \frac{1}{R_1(\Delta t)} \times \frac{C(3\pi^0, \ell^+)}{C(\ell^-, \pi\pi)}. \quad (40)$$

Due to the property (34), the proportionality constant between  $R_{2(4)}^{\text{exp}}(-\Delta t)$  and  $1/R_{3(1)}(\Delta t)$  is the same as the one between  $R_{2(4)}^{\text{exp}}(\Delta t)$  and  $R_{2(4)}(\Delta t)$ . Therefore one can actually measure only two observables,  $R_2^{\text{exp}}(\Delta t)$  and  $R_4^{\text{exp}}(\Delta t)$ , with  $-\infty < \Delta t < +\infty$ ; their expected behavior is shown in Fig.2.

From the point of view of a model independent and direct test of  $\mathcal{T}$  symmetry, it would be sufficient to prove that one of the predictions in eq.(16) is not satisfied, i.e. that  $R_i(\Delta t) \neq 1$ , for any ratio  $R_i$ . Experimentally one can adopt two different strategies to obtain this result:

1. The first one is to observe any significant dependence on  $\Delta t$  in the measured ratio  $R_2^{\text{exp}}(\Delta t)$  or  $R_4^{\text{exp}}(\Delta t)$ ; therefore one may conclude that the corresponding ratio  $R_i$  is not constant and cannot satisfy the prediction in eq.(16).
2. The second strategy consists in measuring the ratio  $R_2^{\text{exp}}(\Delta t)$  or  $R_4^{\text{exp}}(\Delta t)$  in the limit  $\Delta t \gg \tau_S$ , where they are expected to have a constant value; given an independent evaluation of the corresponding ratio of coefficients  $\frac{C(\ell^-, 3\pi^0)}{C(\pi\pi, \ell^+)}$  or  $\frac{C(\ell^+, 3\pi^0)}{C(\pi\pi, \ell^-)}$  one may extract the asymptotic value  $R_2(\Delta t \gg \tau_S)$  or  $R_4(\Delta t \gg \tau_S)$  and verify the predicted deviation from one, eq.(26) or (27).

For the second strategy we can consider that:

$$\begin{aligned} \frac{C(\ell^-, 3\pi^0)}{C(\pi\pi, \ell^+)} &= \left| \frac{\langle \ell^- | T | \bar{K}^0 \rangle \langle 3\pi^0 | T | K_- \rangle}{\langle \ell^+ | T | K^0 \rangle \langle \pi\pi | T | K_+ \rangle} \right|^2 \\ &= \left| \frac{\langle 3\pi^0 | T | K_- \rangle}{\langle \pi\pi | T | K_+ \rangle} \right|^2 \end{aligned} \quad (41)$$

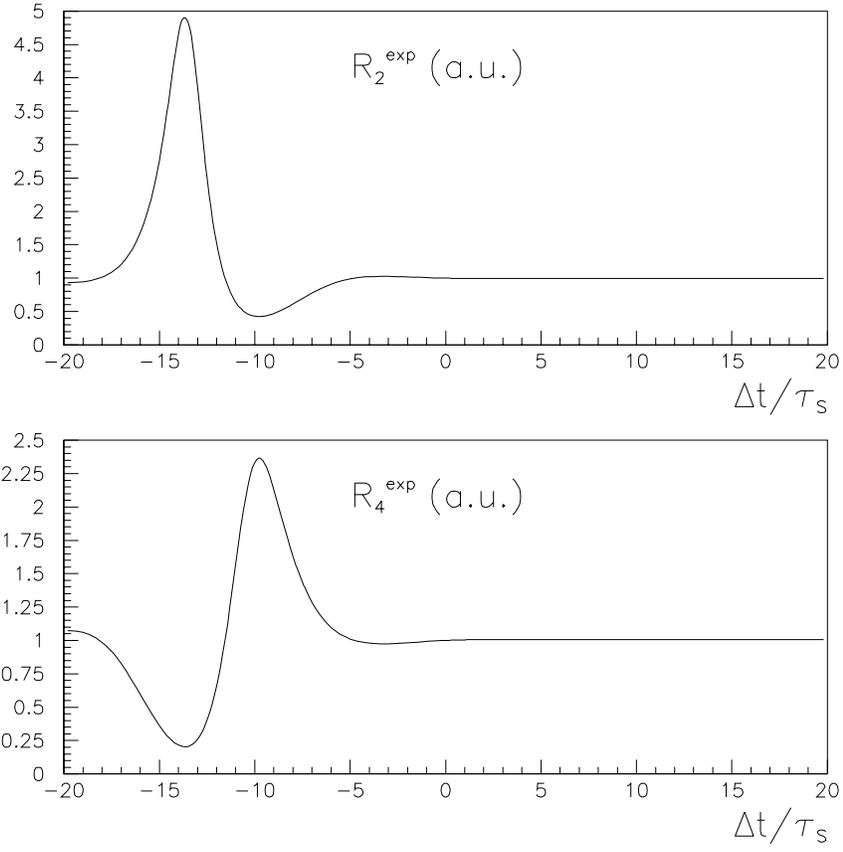


Figure 2: The ratios  $R_2^{\text{exp}}$  and  $R_4^{\text{exp}}$  as a function of  $\Delta t$ .

$$\begin{aligned}
\frac{C(\ell^+, 3\pi^0)}{C(\pi\pi, \ell^-)} &= \left| \frac{\langle \ell^+ | T | \bar{K}^0 \rangle \langle 3\pi^0 | T | K_- \rangle}{\langle \ell^- | T | \bar{K}^0 \rangle \langle \pi\pi | T | K_+ \rangle} \right|^2 \\
&= \left| \frac{\langle 3\pi^0 | T | K_- \rangle}{\langle \pi\pi | T | K_+ \rangle} \right|^2
\end{aligned} \tag{42}$$

neglecting possible  $\mathcal{CP}\mathcal{T}$  violation effects in semileptonic decays.

Neglecting second order terms, one has:

$$\begin{aligned}
\text{BR}(K_S \rightarrow \pi\pi) \Gamma_S &= |\langle \pi\pi | T | K_S \rangle|^2 = \left| \frac{\langle \pi\pi | T | K_+ \rangle}{N_+(1 - \alpha\beta)} \right|^2 \\
&\simeq |\langle \pi\pi | T | K_+ \rangle|^2
\end{aligned} \tag{43}$$

$$\begin{aligned}
\text{BR}(K_L \rightarrow 3\pi^0) \Gamma_L &= |\langle 3\pi^0 | T | K_L \rangle|^2 = \left| \frac{\langle 3\pi^0 | T | K_- \rangle}{N_-(1 - \alpha\beta)} \right|^2 \\
&\simeq |\langle 3\pi^0 | T | K_- \rangle|^2 .
\end{aligned} \tag{44}$$

Using the above relations, eqs.(43) and (44), one has:

$$\frac{C(\ell^-, 3\pi^0)}{C(\pi\pi, \ell^+)} \simeq \frac{C(\ell^+, 3\pi^0)}{C(\pi\pi, \ell^-)} \simeq \frac{\text{BR}(K_L \rightarrow 3\pi^0) \Gamma_L}{\text{BR}(K_S \rightarrow \pi\pi) \Gamma_S} . \tag{45}$$

Therefore in the case of the second strategy, one can evaluate the ratio of coefficients in terms of measurable branching ratios, and convert with the correct normalization the measured ratios  $R_2^{\text{exp}}$  and  $R_4^{\text{exp}}$  into the corresponding values for  $R_2$  and  $R_4$ , making possible a direct comparison of these values with the prediction (16) obtained in the case of  $\mathcal{T}$  symmetry invariance.

One can define the statistical sensitivity of an experiment

$$Q_i(\Delta t) \equiv \frac{|1 - R_i(\Delta t)|}{\sigma(R_i(\Delta t))} , \tag{46}$$

as the ratio between the expected deviation of  $R_i$  from prediction (16), as given by the measured value of  $\epsilon$ , and the statistical uncertainty on  $R_i$ , in a bin width of  $1 \tau_S$  centered at the value  $\Delta t$ , as shown in Figs. 3 and 4 <sup>2</sup>.

---

<sup>2</sup>The plots in Figs. 3 and 4 have been evaluated assuming a large number of counts and Poisson fluctuations in each  $\Delta t$  bin of the measured  $I(f_1, f_2; \Delta t)$  distributions, and negligible uncertainties due to the knowledge of the ratio of coefficients (45) (needed for the second strategy).

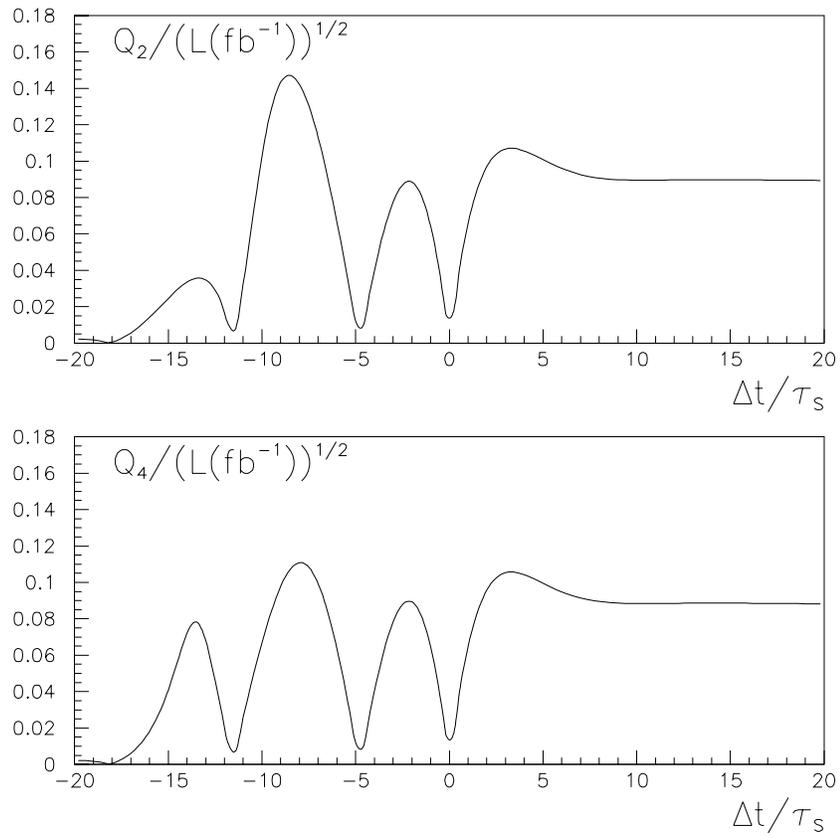


Figure 3: The statistical sensitivity  $Q_2(\Delta t)$  (top) and  $Q_4(\Delta t)$  (bottom) as a function of  $\Delta t$  and normalized to the square root of the integrated luminosity  $\sqrt{L(\text{fb}^{-1})}$ .

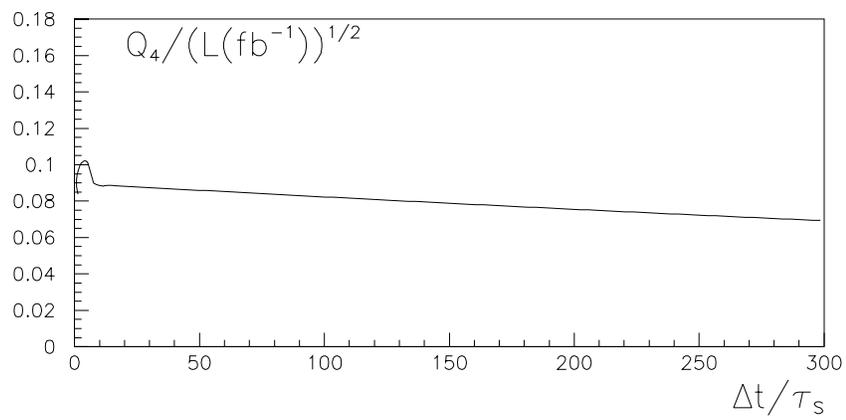
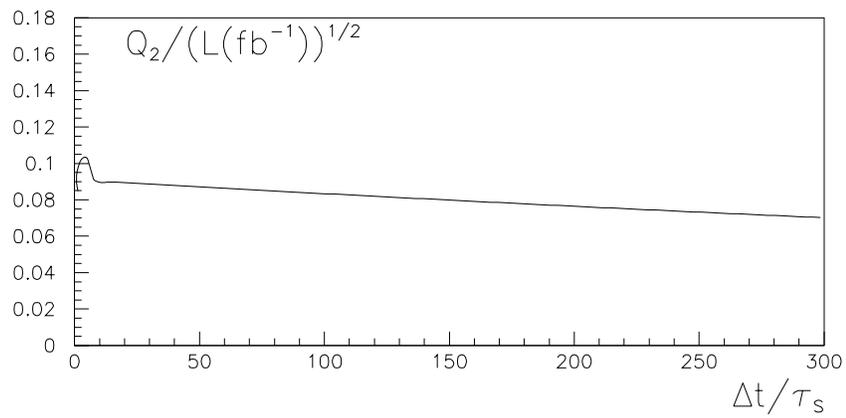


Figure 4: As in Fig.3 but in the range  $0 < \Delta t < 300 \tau_S$ .

It is worth noting that the sensitivity of the test in the region  $\Delta t < 0$  is limited by the large statistical uncertainties on  $R_i$  (due to a fast exponential decrease of the events in this region) despite the expected large deviations of  $R_i$  from unity. On the other hand, in the statistically most populated region at  $\Delta t > 0$ , the sensitivity is not large because small deviations of  $R_i$  from unity are expected here (see eqs.(26) and (27)).

In the case of the KLOE-2 experiment at DAΦNE, where an integrated luminosity  $L$  of  $\mathcal{O}(10 \text{ fb}^{-1})$  is expected [20], the  $I(f_1, f_2; \Delta t)$  distributions have been evaluated with a simple Monte Carlo simulation, making the approximation of a gaussian  $\Delta t$  experimental resolution with  $\sigma = 1 \tau_S$ , and a full detection efficiency, as shown in Fig. 5. It can be noticed that the  $I(\ell^\pm, 3\pi^0; \Delta t)$  distributions have very few or no events for  $\Delta t \lesssim -5 \tau_S$ . While a complete feasibility study is beyond the scope of the present paper, it appears that the first strategy described above is difficult to be implemented at KLOE-2 due to the lack of enough statistics, whereas the second strategy is much more viable. In fact considering a large  $\Delta t$  interval in the statistically most populated region, e.g.  $0 \leq \Delta t \leq 300 \tau_S$ , a much larger global sensitivity of  $Q \simeq 4.4, 6.2$ , and  $8.8$  is obtained for  $L = 5, 10$ , and  $20 \text{ fb}^{-1}$ , respectively.

## 5. Conclusions

It has been shown that, by exploiting the EPR entanglement of neutral kaon pairs produced at a  $\phi$ -factory, it is possible to perform a direct test of the time reversal symmetry in the neutral kaon system, independently from  $\mathcal{CP}$  violation and  $\mathcal{CPT}$  invariance constraints, and therefore overcoming some conceptual difficulties affecting previous tests. The proposed test is highly model-independent, relying only on the validity of quantum mechanical prescriptions and EPR correlations. From the experimental point of view, the test would require to measure ratios of intensities (29) with a suitable choice of decay products in definite time ordering. The absolute normalization of the measured ratios requires the knowledge of measurable kaon branching ratios and lifetimes and would not suffer from other uncertainties. The KLOE-2 experiment at the DAΦNE  $\phi$ -factory could make a significant  $\mathcal{T}$  symmetry test with an integrated luminosity of  $\mathcal{O}(10 \text{ fb}^{-1})$ .

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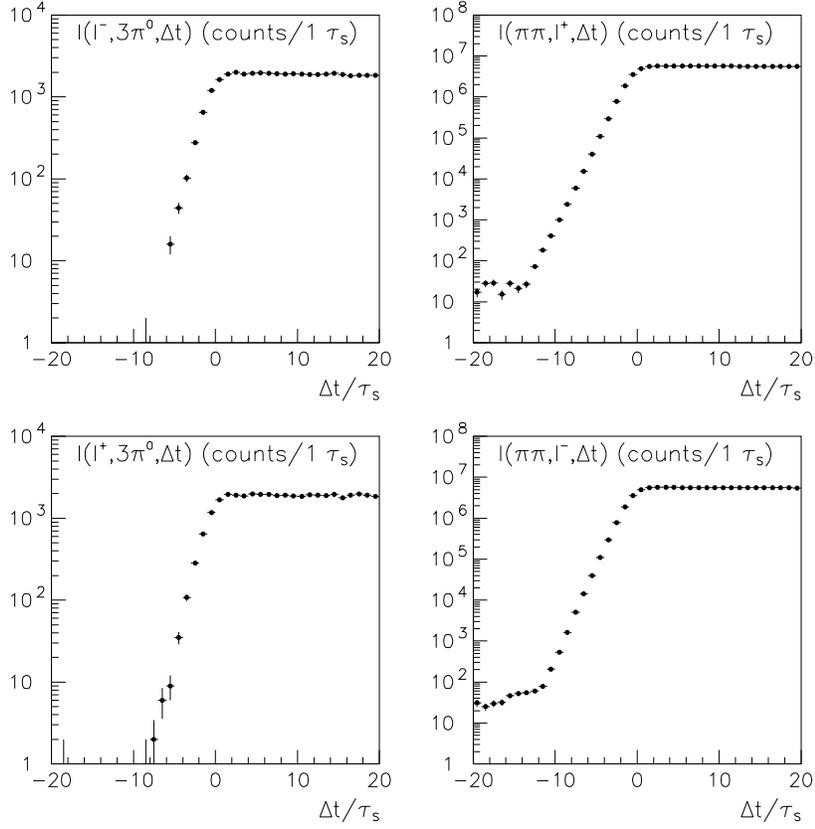


Figure 5: The  $I(\ell^-, 3\pi^0; \Delta t)$  (top left),  $I(\pi\pi, \ell^+; \Delta t)$  (top right),  $I(\ell^+, 3\pi^0; \Delta t)$  (bottom left), and  $I(\pi\pi, \ell^-; \Delta t)$  (bottom right) distributions as a function of  $\Delta t$  evaluated with a simple Monte Carlo simulation, making the approximation of a gaussian  $\Delta t$  experimental resolution with  $\sigma = 1 \tau_S$ , a full detection efficiency, and assuming  $L = 10 \text{ fb}^{-1}$ .

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## Appendix A. Orthogonality constrains

The orthogonality assumption (10) and condition (12) constrain the  $\eta_{\pi\pi}$  and  $(\eta_{3\pi^0}^{-1})$  parameters. Concerning the  $\eta_{\pi\pi}$  parameter one could safely neglect any contribution from direct  $\mathcal{CP}$  violation, because  $(\epsilon'/\epsilon)$  is experimentally known to be  $\mathcal{O}(10^{-3})$  [6]. One can also safely neglect possible contributions from direct  $\mathcal{CPT}$  violation in the  $\pi\pi$  decay. Therefore for the purposes

of the present test, one can assume  $\eta_{\pi\pi} \simeq \epsilon_L$  (e.g. adopting the Wu-Yang phase convention).

Even though it would be reasonable to expect also for the  $(\eta_{3\pi^0}^{-1})$  parameter a negligible contribution from direct  $\mathcal{CP}$  and  $\mathcal{CP}\mathcal{T}$  violations [27, 28], i.e.  $(\eta_{3\pi^0}^{-1}) \simeq \epsilon_S$ , unfortunately the experimental knowledge on this parameter is much less precise than for  $\eta_{\pi\pi}$ , resulting at present in an upper limit  $(\eta_{3\pi^0}^{-1}) < 9 \times 10^{-3}$  at 90% C.L. [29]. However, also assuming a contribution from direct  $\mathcal{CP}$  violation much larger than in the case of  $\pi\pi$ , e.g. giving rise to a  $\pm 10\%$  variation in the absolute value of  $(\eta_{3\pi^0}^{-1})$ , or a  $\pm 10^\circ$  variation of its phase (with respect to the expected value, i.e.  $(\eta_{3\pi^0}^{-1}) \simeq \epsilon_S \simeq \epsilon$ ), the impact of these variations on the measured ratios  $R_i^{\text{exp}}(\Delta t)$  does not spoil the significance of the  $\mathcal{T}$  symmetry test in the  $\Delta t$  region statistically relevant for the KLOE-2 experiment at DAΦNE, i.e.  $\Delta t \gtrsim -5\tau_S$ , as shown in Figs. A.6 and A.7, where  $|\langle 3\pi^0 | T | K_L \rangle|^2$  has been kept fixed to its measured value [6] while varying  $(\eta_{3\pi^0}^{-1})$ . Thus one can conclude that direct  $\mathcal{CP}$  violation can be safely neglected.

Apart from these considerations, it is also possible to experimentally perform a direct test of assumption (10) by measuring the ratio of processes  $K^0 \rightarrow K_+$  vs.  $\tilde{K}_+ \rightarrow \bar{K}^0$ . In fact, taking into account the difference between the tagged state  $\tilde{K}_+$  and the decaying state  $K_+$ , using eq.(21) one can easily evaluate the following ratio:

$$\frac{P[K^0(0) \rightarrow K_+(\Delta t)]}{P[\tilde{K}_+(0) \rightarrow \bar{K}^0(\Delta t)]} \simeq \frac{\left| e^{-i\lambda_S \Delta t} \left( \frac{1 - \epsilon_L}{\sqrt{2}} \right) + e^{-i\lambda_L \Delta t} (\eta_{\pi\pi}) \left( \frac{1 - \epsilon_S}{\sqrt{2}} \right) \right|^2}{\left| e^{-i\lambda_S \Delta t} \left( \frac{1 - \epsilon_S}{\sqrt{2}} \right) + e^{-i\lambda_L \Delta t} (\eta_{3\pi^0}^{-1}) \left( \frac{1 - \epsilon_L}{\sqrt{2}} \right) \right|^2}, \quad (\text{A.1})$$

which is constrained to be 1 if the condition  $\eta_{\pi\pi} = (\eta_{3\pi^0}^{-1})$  holds, with the assumption of  $\mathcal{CP}\mathcal{T}$  invariance ( $\epsilon_S = \epsilon_L = \epsilon$ ). Thus measuring this ratio with enough precision, one can evaluate whether the direct  $\mathcal{CP}$  violation contribution to the  $3\pi^0$  decay is negligible, or not. Analogous considerations apply to other ratios like:

- $P[\bar{K}^0(0) \rightarrow K_+(\Delta t)]/P[\tilde{K}_+(0) \rightarrow K^0(\Delta t)]$
- $P[K^0(0) \rightarrow K_-(\Delta t)]/P[\tilde{K}_-(0) \rightarrow \bar{K}^0(\Delta t)]$
- $P[\bar{K}^0(0) \rightarrow K_-(\Delta t)]/P[\tilde{K}_-(0) \rightarrow K^0(\Delta t)]$

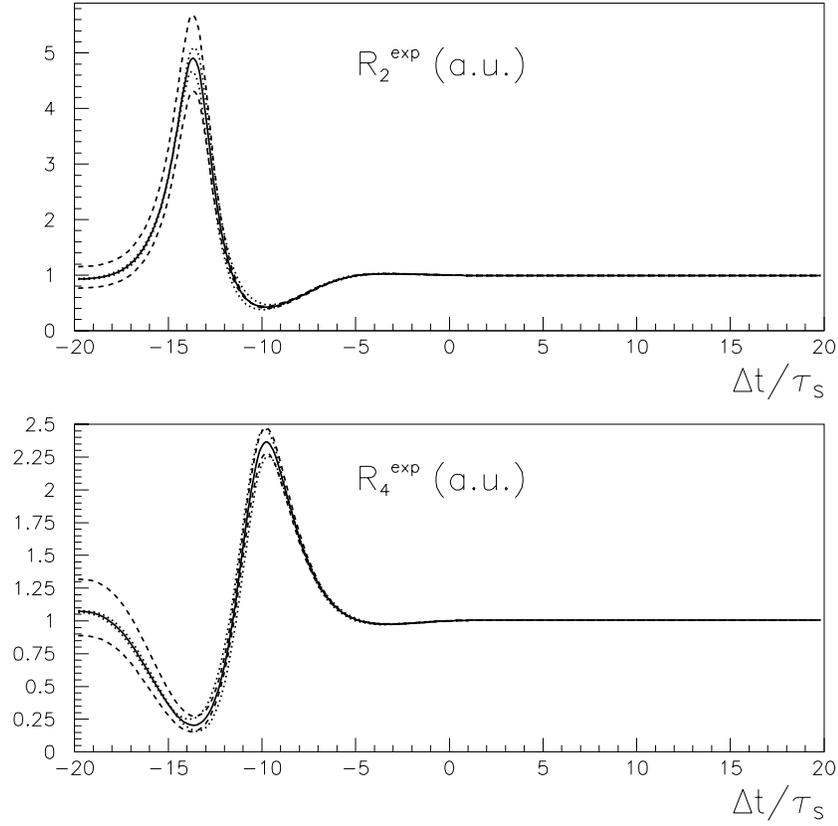


Figure A.6: The expected ratios  $R_2^{\text{exp}}(\Delta t)$  (top) and  $R_4^{\text{exp}}(\Delta t)$  (bottom) as a function of  $\Delta t$  (solid line); dashed lines correspond to  $\pm 10\%$  variation in the absolute value of  $(\eta_{3\pi^0}^{-1})$ , while dotted lines correspond to a  $\pm 10^\circ$  variation of its phase (with respect to the expected value, i.e.  $(\eta_{3\pi^0}^{-1}) \simeq \epsilon_S \simeq \epsilon$ ). The value of  $|\langle 3\pi^0 | T | K_L \rangle|^2$  has been kept fixed while varying  $(\eta_{3\pi^0}^{-1})$ .

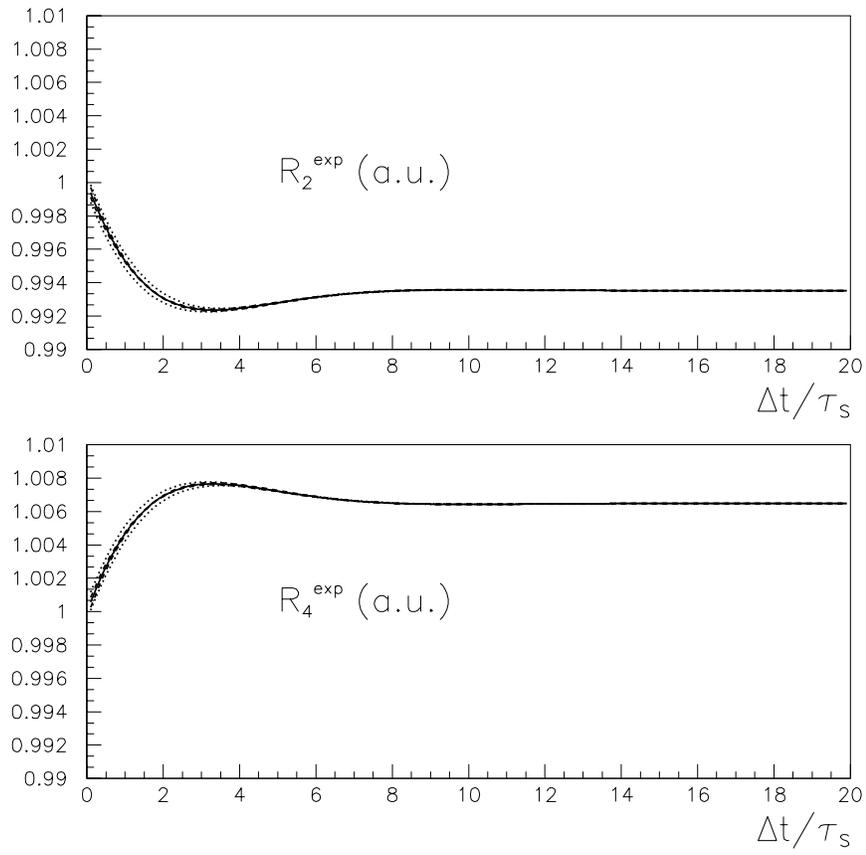


Figure A.7: A zoom of the plots shown in fig.A.6 in the region  $0 \leq \Delta t \leq 20\tau_S$ , which is statistically relevant for the KLOE-2 experiment at DAΦNE.